

5 Machten, exponenten en logaritmen

Voorkennis Herleiden van machten

Bladzijde 10

- 1** a $x^2 \cdot x^3 = x^5$
 b $2p^3 \cdot 3p^2 = 6p^5$
 c $4a^2b \cdot 5a^3b^2 = 20a^5b^3$
- 2** a $(p^2q)^3 = p^6q^3$
 b $(3x^2)^3 = 27x^6$
 c $(-5x^2y^3)^2 = 25x^4y^6$
- 3** a $\frac{12x^6}{4x^2} = 3x^4$
 b $\frac{5x^{10}}{15x^5} = \frac{x^5}{3} = \frac{1}{3}x^5$
 c $\frac{24a^4b^2}{6ab} = 4a^3b$
- 4** a $\frac{-28a^6}{7a} = -4a^5$
 b $-(3a^4)^2 = -9a^8$
 c $(-2a^2)^5 = -32a^{10}$
- 5** a $(ab)^4 \cdot a = a^4b^4 \cdot a = a^5b^4$
 b $(-2ab)^3 \cdot b = -8a^3b^3 \cdot b = -8a^3b^4$
 c $(3a)^2 + (2b)^2 = 9a^2 + 4b^2$
- 6** a $a^{2n} \cdot a^{n-1} = a^{2n+n-1} = a^{3n-1}$
 b $a^{n^2-1} \cdot a^{n-1} = a^{n^2-1+n-1} = a^{n^2+n-2}$
 c $\frac{a^{n^2-n}}{a^{n-1}} = a^{n^2-n-(n-1)} = a^{n^2-n-n+1} = a^{n^2-2n+1}$
- d** $-2p^4q^3 \cdot -3pq = 6p^5q^4$
e $5x^2y \cdot 2x - 3x^3y = 10x^3y - 3x^3y = 7x^3y$
f $12a^4b \cdot \frac{1}{4}ab - 8ab = 3a^5b^2 - 8ab$
- d** $(-4ab^4)^2 = 16a^2b^8$
e $(3a)^2 \cdot (2a^2)^3 = 9a^2 \cdot 8a^6 = 72a^8$
f $(3a^3)^2 + (2a^2)^3 = 9a^6 + 8a^6 = 17a^6$
- d** $\frac{-15p^6q}{5p^2q} = -3p^4$
e $\frac{10x^3y^2}{5x^2y} = 2xy$
f $\frac{(2ab)^3}{(3ab)^2} = \frac{8a^3b^3}{9a^2b^2} = \frac{8}{9}ab$
- d** $(-a^3)^3 = -a^9$
e $(5a)^3 \cdot -3a = 125a^3 \cdot -3a = -375a^4$
f $\left(\frac{9a^4}{a}\right)^2 = (9a^3)^2 = 81a^6$
- d** $(3a)^3 - 8a^3 = 27a^3 - 8a^3 = 19a^3$
e $\left(\frac{1}{2}a\right)^2 + (-a)^2 = \frac{1}{4}a^2 + a^2 = 1\frac{1}{4}a^2$
f $(5a^4)^2 + (-a^2)^4 = 25a^8 + a^8 = 26a^8$

5.1 Machten met negatieve en gebroken exponenten

Bladzijde 11

- 1** a De exponenten worden telkens 1 minder en de getallen worden steeds door 2 gedeeld,
 dus $2^1 = \frac{4}{2} = 2$, $2^0 = \frac{2}{2} = 1$, $2^{-1} = \frac{1}{2}$ en $2^{-2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$.
- b** $2^{-3} = \frac{1}{2^3}$
 $2^{-4} = \frac{1}{2^4}$
- c** $x^0 = 1$
 $x^{-1} = \frac{1}{x}$
 $x^{-3} = \frac{1}{x^3}$

Bladzijde 12

2 a $\frac{1}{a^2} = a^{-2}$ f $\frac{\left(\frac{1}{a^5}\right)}{a} = \frac{a^{-5}}{a^1} = a^{-5-1} = a^{-6}$
 b $a^4 \cdot \frac{1}{a^6} = a^4 \cdot a^{-6} = a^{-2}$ g $\frac{a}{a^{12}} = \frac{a^1}{a^{12}} = a^{1-12} = a^{-11}$
 c $\frac{a^n}{\left(\frac{1}{a^4}\right)} = \frac{a^n}{a^{-4}} = a^{n+4}$ h $\frac{1}{a^8} \cdot (a^3)^n = a^{-8} \cdot a^{3n} = a^{-8+3n}$
 d $\frac{a^8}{a^0} = a^{8-0} = a^8$ i $\frac{\left(\frac{1}{a^n}\right)}{a^{-3}} = \frac{a^{-n}}{a^{-3}} = a^{-n-(-3)} = a^{-n+3}$
 e $(a^3)^{-2} = a^{-6}$

Bladzijde 13

3 a $6a^{-5}b^3 = 6 \cdot \frac{1}{a^5} \cdot b^3 = \frac{6b^3}{a^5}$
 b $\frac{1}{3}a^{-3} = \frac{1}{3} \cdot \frac{1}{a^3} = \frac{1}{3a^3}$
 c $(5a^{-4}b^2)^{-1} = 5^{-1} \cdot (a^{-4})^{-1} \cdot (b^2)^{-1} = \frac{1}{5}a^4b^{-2} = \frac{a^4}{5b^2}$
 d $\frac{3}{5}a^{-4} = \frac{3}{5} \cdot \frac{1}{a^4} = \frac{3}{5a^4}$
 e $\left(\frac{1}{2}a\right)^{-3} = (2^{-1} \cdot a)^{-3} = 2^3 \cdot a^{-3} = 8 \cdot \frac{1}{a^3} = \frac{8}{a^3}$
 f $\frac{1}{6}a^{-2}b^4 = \frac{1}{6} \cdot \frac{1}{a^2} \cdot b^4 = \frac{b^4}{6a^2}$
 g $-4 \cdot (3a)^{-2} = -4 \cdot \frac{1}{(3a)^2} = -4 \cdot \frac{1}{9a^2} = -\frac{4}{9a^2}$
 h $(3a)^{-2}b^{-3} = \frac{1}{(3a)^2} \cdot \frac{1}{b^3} = \frac{1}{9a^2b^3}$
 i $\left(\frac{3}{8}a^{-1}b\right)^{-2} = \left(\frac{3}{8}\right)^{-2} \cdot (a^{-1})^{-2} \cdot b^{-2} = \left(\frac{8}{3}\right)^2 \cdot a^2 \cdot \frac{1}{b^2} = \frac{64}{9} \cdot a^2 \cdot \frac{1}{b^2} = \frac{64a^2}{9b^2}$

4 Uit $(\sqrt[3]{x})^3 = x$ en $(x^{\frac{1}{3}})^3 = x$ volgt $x^{\frac{1}{3}} = \sqrt[3]{x}$.

Bladzijde 14

5 a $a \cdot \sqrt[3]{a} = a^1 \cdot a^{\frac{1}{3}} = a^{1\frac{1}{3}}$ f $\sqrt[3]{\frac{1}{a^2}} = \sqrt[3]{a^{-2}} = a^{-\frac{2}{3}}$
 b $\frac{1}{\sqrt{a}} = \frac{1}{a^{\frac{1}{2}}} = a^{-\frac{1}{2}}$ g $\sqrt[3]{a^{12}} = a^{\frac{12}{3}} = a^4$
 c $\frac{1}{a} = \frac{1}{a^1} = a^{-1}$ h $\frac{1}{a^4} \cdot \sqrt[3]{a} = a^{-4} \cdot a^{\frac{1}{3}} = a^{-4+\frac{1}{3}} = a^{-\frac{11}{3}}$
 d $\frac{1}{a^3} = a^{-3}$ i $\frac{a^3}{\sqrt[3]{a}} = \frac{a^3}{a^{\frac{1}{3}}} = a^{3-\frac{1}{3}} = a^{\frac{8}{3}}$
 e $a^2 \cdot \sqrt{a} = a^2 \cdot a^{\frac{1}{2}} = a^{2\frac{1}{2}}$

6 a $8\sqrt{2} = 2^3 \cdot 2^{\frac{1}{2}} = 2^{3\frac{1}{2}}$
 b $\frac{1}{3}\sqrt{3} = 3^{-1} \cdot 3^{\frac{1}{2}} = 3^{-1+\frac{1}{2}} = 3^{-\frac{1}{2}}$
 c $\frac{4\sqrt{2}}{\sqrt[3]{2}} = \frac{2^2 \cdot 2^{\frac{1}{2}}}{2^{\frac{1}{3}}} = 2^{2+\frac{1}{2}-\frac{1}{3}} = 2^{2\frac{1}{6}}$
 d $\frac{1}{100}\sqrt{10} = 10^{-2} \cdot 10^{\frac{1}{2}} = 10^{-2+\frac{1}{2}} = 10^{-1\frac{1}{2}}$
 e $\frac{1}{8} \cdot \sqrt[3]{\frac{1}{4}} = \frac{1}{2^3} \cdot \sqrt[3]{2^{-2}} = 2^{-3} \cdot 2^{-\frac{2}{3}} = 2^{-3-\frac{2}{3}} = 2^{-\frac{11}{3}}$
 f $10 \cdot \sqrt[3]{0,1} = 10^1 \cdot \sqrt[3]{10^{-1}} = 10^1 \cdot 10^{-\frac{1}{3}} = 10^{1-\frac{1}{3}} = 10^{\frac{2}{3}}$

7 a $5a^{\frac{2}{3}} = 5a^3 \cdot a^{\frac{1}{3}} = 5a^3 \cdot \sqrt[3]{a}$

b $\frac{1}{2}a^{-\frac{1}{4}}b = \frac{b}{2a^{\frac{1}{4}}} = \frac{b}{2 \cdot \sqrt[4]{a}}$

c $3a^{-\frac{2}{3}} = \frac{3}{a^{\frac{2}{3}}} = \frac{3}{\sqrt[3]{a^2}}$

d $\frac{2}{3}a^{-3} \cdot b^{\frac{1}{2}} = \frac{2b^{\frac{1}{2}}}{3a^3} = \frac{2b\sqrt{b}}{3a^3}$

e $\frac{1}{5}a^{-\frac{1}{2}} \cdot b^{\frac{1}{3}} = \frac{b^{\frac{1}{3}}}{5a^{\frac{1}{2}}} = \frac{\sqrt[3]{b}}{5\sqrt{a}}$

f $(5a)^{-\frac{1}{2}} = \frac{1}{(5a)^{\frac{1}{2}}} = \frac{1}{\sqrt{5a}}$

8 a $\frac{x^6}{x^2 \cdot \sqrt{x}} = \frac{x^6}{x^2 \cdot x^{\frac{1}{2}}} = \frac{x^6}{x^{\frac{5}{2}}} = x^{6-2\frac{1}{2}} = x^{3\frac{1}{2}}$

b $x \cdot \sqrt[7]{x^3} = x^1 \cdot x^{\frac{3}{7}} = x^{1\frac{3}{7}}$

c $\frac{x}{\sqrt[5]{x}} = \frac{x^1}{x^{\frac{1}{5}}} = x^{1-\frac{1}{5}} = x^{\frac{4}{5}}$

d $x^4 \cdot \sqrt{x} = x^4 \cdot x^{\frac{1}{2}} = x^{4\frac{1}{2}}$

e $\frac{\sqrt[3]{x}}{\sqrt{x}} = \frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{\frac{1}{3}-\frac{1}{2}} = x^{-\frac{1}{6}}$

f $\frac{x^4 \cdot \sqrt[5]{x}}{x^5 \cdot \sqrt[4]{x}} = \frac{x^4 \cdot x^{\frac{1}{5}}}{x^5 \cdot x^{\frac{1}{4}}} = \frac{x^{4\frac{1}{5}}}{x^{5\frac{1}{4}}} = x^{4\frac{1}{5}-5\frac{1}{4}} = x^{-1\frac{1}{20}}$

9 De getallen als macht van 2 geschreven zijn: $2^0, 2^1, 2^{-1}, 2^2, 2^{-3}, 2^5, 2^{-8}, 2^{13}, 2^{-21}, 2^{34}, \dots$
Het tiende getal is dus 2^{34} .

10 a $y = (2x^2)^3 \cdot \frac{2}{x^{10}} = 8x^6 \cdot 2x^{-10} = 16x^{-4}$
Dus $y = 16x^{-4}$.

b $y = \frac{3}{x} \cdot \sqrt[4]{x^3} = 3x^{-1} \cdot x^{\frac{3}{4}} = 3x^{-1+\frac{3}{4}} = 3x^{-\frac{1}{4}}$
Dus $y = 3x^{-\frac{1}{4}}$.

c $y = 3(\frac{1}{3}x^2)^{-2} \cdot 6x^2 = 3 \cdot (\frac{1}{3})^{-2} \cdot (x^2)^{-2} \cdot 6x^2 = 3 \cdot 9 \cdot x^{-4} \cdot 6x^2 = 162x^{-2}$
Dus $y = 162x^{-2}$.

d $y = \frac{5}{3x\sqrt{x}} = \frac{5}{3} \cdot \frac{1}{x \cdot x^{\frac{1}{2}}} = \frac{5}{3} \cdot \frac{1}{x^{1\frac{1}{2}}} = \frac{5}{3}x^{-1\frac{1}{2}}$
Dus $y = \frac{5}{3}x^{-1\frac{1}{2}}$.

11 a $y = \frac{5}{x\sqrt{x}} = \frac{5}{x^{1\frac{1}{2}}} = 5x^{-1\frac{1}{2}}$
Dus $y = 5x^{-1\frac{1}{2}}$.

b $y = 5x\sqrt{x^3} = 5x^1 \cdot x^{\frac{3}{2}} = 5x^{1+\frac{3}{2}} = 5x^{2\frac{1}{2}}$
Dus $y = 5x^{2\frac{1}{2}}$.

c $y = \frac{5}{x^3} \cdot 2\sqrt{x} = 5 \cdot x^{-3} \cdot 2x^{\frac{1}{2}} = 10x^{-3+\frac{1}{2}} = 10x^{-2\frac{1}{2}}$
Dus $y = 10x^{-2\frac{1}{2}}$.

d $y = 72x(\frac{1}{4}x\sqrt{x})^3 = 72x \cdot (\frac{1}{4})^3 \cdot (x^{1\frac{1}{2}})^3 = 72x \cdot \frac{1}{64} \cdot x^{4\frac{1}{2}} = \frac{9}{8}x^{5\frac{1}{2}}$
Dus $y = \frac{9}{8}x^{5\frac{1}{2}}$.

Bladzijde 15

12 a $(x^{\frac{2}{3}})^{\frac{3}{2}} = 10^{\frac{3}{2}}$

b $(x^{\frac{2}{3}})^{\frac{3}{2}} = 10^{\frac{3}{2}}$
 $x^1 = 10^1 \cdot 10^{\frac{1}{2}}$
 $x = 10\sqrt{10}$

$$\begin{aligned} 13 \quad a \quad x^{\frac{2}{3}} &= 9 \\ x &= 9^{\frac{3}{2}} \\ x &= 9\sqrt{9} = 27 \end{aligned}$$

$$\begin{aligned} b \quad 8x^{-\frac{1}{2}} &= 1 \\ x^{-\frac{1}{2}} &= \frac{1}{8} \\ x &= \left(\frac{1}{8}\right)^{-\frac{2}{1}} \\ x &= (2^{-3})^{-\frac{2}{1}} \\ x &= 2^2 = 4 \end{aligned}$$

$$\begin{aligned} c \quad 5 - 2x^{-3} &= 4 \\ -2x^{-3} &= -1 \\ x^{-3} &= \frac{1}{2} \\ x &= \left(\frac{1}{2}\right)^{-\frac{1}{3}} \\ x &= (2^{-1})^{-\frac{1}{3}} \\ x &= 2^{\frac{1}{3}} = \sqrt[3]{2} \\ d \quad \sqrt{(2x)^3} &= \frac{1}{8} \\ (2x)^3 &= \left(\frac{1}{8}\right)^2 \\ 8x^3 &= \frac{1}{8^2} \\ x^3 &= \frac{1}{8^3} \\ x &= \frac{1}{8} \end{aligned}$$

Bladzijde 16

$$\begin{aligned} 14 \quad a \quad \frac{1}{3}x^{\frac{1}{2}} &= 2\frac{2}{3} - \frac{1}{3}x^{\frac{1}{2}} \\ \frac{2}{3}x^{\frac{1}{2}} &= 2\frac{2}{3} \\ x^{\frac{1}{2}} &= 4 \\ x &= 4^2 = \sqrt[3]{16} \end{aligned}$$

$$\begin{aligned} b \quad 3 \cdot \sqrt[4]{(2x)^{-1}} &= 6 \\ \sqrt[4]{(2x)^{-1}} &= 2 \\ (2x)^{-\frac{1}{4}} &= 2 \\ 2x &= 2^{-4} \\ 2x &= \frac{1}{16} \\ x &= \frac{1}{32} \end{aligned}$$

$$\begin{aligned} c \quad (x+9)^{-\frac{1}{2}} &= \frac{8}{27} \\ x+9 &= \left(\frac{8}{27}\right)^{-2} \\ x+9 &= \left(\left(\frac{2}{3}\right)^3\right)^{-2} \\ x+9 &= \left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{1}{\frac{4}{9}} = \frac{9}{4} \\ x &= -6\frac{3}{4} \end{aligned}$$

$$\begin{aligned} d \quad \frac{1}{3}(x^2-3)^{2\frac{1}{2}} &= 81 \\ (x^2-3)^{2\frac{1}{2}} &= 243 \\ x^2-3 &= 243^{\frac{2}{5}} = (3^5)^{\frac{2}{5}} = 3^2 = 9 \\ x^2 &= 12 \\ x &= \sqrt{12} \vee x = -\sqrt{12} \\ x &= 2\sqrt{3} \vee x = -2\sqrt{3} \end{aligned}$$

$$\begin{aligned} 15 \quad (x^2+10)^{\frac{1}{2}} &= 3x^2(x^2+10)^{\frac{1}{2}} \\ (x^2+10)^1 \cdot (x^2+10)^{\frac{1}{2}} &= 3x^2(x^2+10)^{\frac{1}{2}} \\ (x^2+10)^{\frac{1}{2}} &= 0 \vee x^2+10 = 3x^2 \\ x^2+10 &= 0 \vee 2x^2 = 10 \\ x^2 &= -10 \vee x^2 = 5 \\ x &= \sqrt{5} \vee x = -\sqrt{5} \end{aligned}$$

$$\begin{aligned} 16 \quad y &= 27x^3 \\ 27x^3 &= y \\ x^3 &= \frac{1}{27}y \\ x &= \left(\frac{1}{27}y\right)^{\frac{1}{3}} \\ x &= \left(\frac{1}{27}\right)^{\frac{1}{3}} \cdot y^{\frac{1}{3}} \\ x &= (3^{-3})^{\frac{1}{3}} \cdot y^{\frac{1}{3}} \\ x &= 3^{-1} \cdot y^{\frac{1}{3}} \\ x &= \frac{1}{3} \cdot y^{\frac{1}{3}} \\ \text{Dus } x &= \frac{1}{3}y^{\frac{1}{3}}. \end{aligned}$$

Bladzijde 17

17 a $y = 5x^{1\frac{1}{2}}$
 $5x^{1\frac{1}{2}} = y$
 $x^{\frac{3}{2}} = 0,2y$
 $x = (0,2y)^{\frac{2}{3}}$
 $x = 0,2^{\frac{2}{3}} \cdot y^{\frac{2}{3}}$
 $x \approx 0,34 \cdot y^{0,67}$
Dus $x = 0,34y^{0,67}$.

b $y = 0,1x^{-1\frac{2}{3}}$
 $0,1x^{-1\frac{2}{3}} = y$
 $x^{-\frac{5}{3}} = 10y$
 $x = (10y)^{-\frac{3}{5}}$
 $x = 10^{-\frac{3}{5}} \cdot y^{-\frac{3}{5}}$
 $x \approx 0,25 \cdot y^{-0,6}$
Dus $x = 0,25y^{-0,6}$.

c $y = 125x^{-2\frac{1}{2}}$
 $125x^{-2\frac{1}{2}} = y$
 $x^{-\frac{5}{2}} = 0,008y$
 $x = (0,008y)^{-\frac{2}{5}}$
 $x = 0,008^{-\frac{2}{5}} \cdot y^{-\frac{2}{5}}$
 $x \approx 6,90 \cdot y^{-0,4}$
Dus $x = 6,90y^{-0,4}$.

18 a $y = 15x \cdot \sqrt[3]{x} = 15x \cdot x^{\frac{1}{3}} = 15x^{1\frac{1}{3}}$
 $y = 15x^{1\frac{1}{3}}$ geeft $15x^{1\frac{1}{3}} = y$
 $x^{\frac{4}{3}} = \frac{1}{15}y$
 $x = (\frac{1}{15}y)^{\frac{3}{4}}$
 $x = (\frac{1}{15})^{\frac{3}{4}} \cdot y^{\frac{3}{4}}$
 $x \approx 0,13 \cdot y^{0,75}$
Dus $x = 0,13y^{0,75}$.

b $y = \frac{12}{x \cdot \sqrt[4]{x}} = \frac{12}{x \cdot x^{\frac{1}{4}}} = \frac{12}{x^{1\frac{1}{4}}} = 12x^{-1\frac{1}{4}}$
 $y = 12x^{-1\frac{1}{4}}$ geeft $12x^{-1\frac{1}{4}} = y$
 $x^{-\frac{5}{4}} = \frac{1}{12}y$
 $x = (\frac{1}{12}y)^{-\frac{4}{5}}$
 $x = (\frac{1}{12})^{-\frac{4}{5}} \cdot y^{-\frac{4}{5}}$
 $x \approx 7,30 \cdot y^{-0,8}$
Dus $x = 7,30y^{-0,8}$.

$$\begin{aligned} \text{c } y &= \frac{6}{x^2 \cdot \sqrt[5]{x^3}} = \frac{6}{x^2 \cdot x^{\frac{3}{5}}} = \frac{6}{x^{2\frac{3}{5}}} = 6x^{-2\frac{3}{5}} \\ y &= 6x^{-2\frac{3}{5}} \text{ geeft } 6x^{-2\frac{3}{5}} = y \\ x^{-\frac{13}{5}} &= \frac{1}{6}y \\ x &= \left(\frac{1}{6}y\right)^{-\frac{5}{13}} \\ x &= \left(\frac{1}{6}\right)^{-\frac{5}{13}} \cdot y^{-\frac{5}{13}} \\ x &\approx 1,99 \cdot y^{-0,38} \\ \text{Dus } x &= 1,99y^{-0,38}. \end{aligned}$$

$$\begin{aligned} \text{19 a } K &= 15q^{-1,6} \text{ geeft } 15q^{-1,6} = K \\ q^{-\frac{8}{5}} &= \frac{1}{15}K \\ q &= \left(\frac{1}{15}K\right)^{-\frac{5}{8}} \\ q &= \left(\frac{1}{15}\right)^{-\frac{5}{8}} \cdot K^{-\frac{5}{8}} \\ q &\approx 5,43 \cdot K^{-0,625} \end{aligned}$$

$$\text{Dus } q = 5,43K^{-0,625}.$$

$$\begin{aligned} \text{b } v &= 25t\sqrt{t} = 25t^{1\frac{1}{2}} \\ v &= 25t^{1\frac{1}{2}} \text{ geeft } 25t^{1\frac{1}{2}} = v \\ t^{\frac{3}{2}} &= 0,04v \\ t &= (0,04v)^{\frac{2}{3}} \\ t &= (0,04)^{\frac{2}{3}} \cdot v^{\frac{2}{3}} \\ t &\approx 0,12 \cdot v^{0,67} \\ \text{Dus } t &= 0,12v^{0,67}. \end{aligned}$$

$$\begin{aligned} \text{20 } F &= \frac{m\sqrt{m}}{m\sqrt{m} - 1} \\ F(m\sqrt{m} - 1) &= m\sqrt{m} \\ m\sqrt{m} \cdot F - F &= m\sqrt{m} \\ m\sqrt{m} \cdot F - m\sqrt{m} &= F \\ m\sqrt{m}(F - 1) &= F \\ m\sqrt{m} &= \frac{F}{F - 1} \\ m^{\frac{3}{2}} &= \frac{F}{F - 1} \\ m &= \left(\frac{F}{F - 1}\right)^{\frac{2}{3}} \end{aligned}$$

Bladzijde 18

$$\begin{aligned} \text{21 a } h_0 &= 0,6 \text{ en } d_0 = 1000 \text{ geeft} \\ h &= 0,6 \cdot \left(\frac{1000}{d}\right)^{0,25} = 0,6 \cdot (1000d^{-1})^{0,25} = 0,6 \cdot 1000^{0,25} \cdot d^{-0,25} \approx 3,37 \cdot d^{-0,25} \\ \text{Dus } h &= 3,37d^{-0,25}. \\ \text{b } d &= 300 \text{ geeft } h = 3,37 \cdot 300^{-0,25} \approx 0,8 \\ \text{De waterhoogte bij een waterdiepte van 300 meter is 8 dm.} \end{aligned}$$

c $h = 3,37d^{-0,25}$ geeft $3,37d^{-0,25} = h$

$$d^{-\frac{1}{4}} = \frac{1}{3,37}h$$

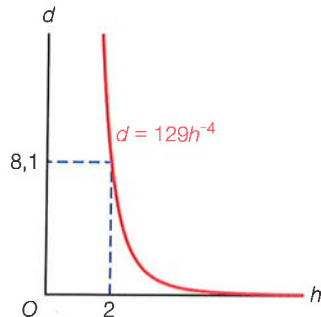
$$d = \left(\frac{1}{3,37}h\right)^{-4}$$

$$d = \left(\frac{1}{3,37}\right)^{-4} \cdot h^{-4}$$

$$d \approx 129 \cdot h^{-4}$$

Dus $d = 129h^{-4}$.

d $h = 2$ geeft $d = 129 \cdot 2^{-4} \approx 8,1$



$h > 2$ geeft $d < 8,1$

Dus bij waterdieptes minder dan 8,1 meter is de golf meer dan 2 meter hoog.

22 a $h = 1,5$ geeft $D = 1014(1 - 0,0226 \cdot 1,5)^{5,26} \approx 846$

Dus op een hoogte van 1,5 km is de luchtdruk 846 mbar.

b $D = 1014(1 - 0,0226h)^{5,26}$ geeft $1014(1 - 0,0226h)^{5,26} = D$

$$(1 - 0,0226h)^{5,26} = \frac{1}{1014}D$$

$$1 - 0,0226h = \left(\frac{1}{1014}D\right)^{\frac{1}{5,26}}$$

$$-0,0226h = \left(\frac{1}{1014}\right)^{\frac{1}{5,26}} \cdot D^{\frac{1}{5,26}} - 1$$

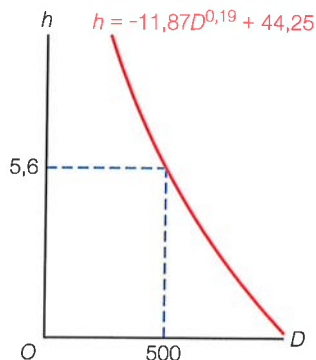
$$h = \frac{\left(\frac{1}{1014}\right)^{\frac{1}{5,26}} \cdot D^{\frac{1}{5,26}} - 1}{-0,0226}$$

$$h \approx -11,87 \cdot D^{0,19} + 44,25$$

Dus $h = -11,87D^{0,19} + 44,25$.

c $0,5 \text{ bar} = 500 \text{ mbar}$

$D = 500$ geeft $h = -11,87 \cdot 500^{0,19} + 44,25 \approx 5,6$



$D < 500$ geeft $h > 5,6$

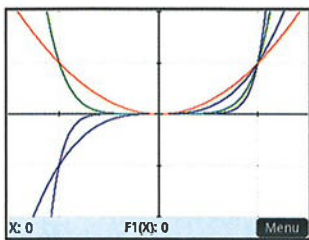
Bij hoogten hoger dan 5,6 km is de luchtdruk minder dan 500 mbar = 0,5 bar.

5.2 Machtsfuncties en wortelfuncties

Bladzijde 20

23

a



b (0, 0) en (1, 1)

c Van de grafieken van f en h ligt geen enkel punt onder de x -as.

Bladzijde 22

24

a

$$y = -\frac{1}{2}x^3$$

↓ translatie $(-3, -5)$

$$y = -\frac{1}{2}(x+3)^3 - 5$$

↓ verm. x -as, -3

$$y = -3(-\frac{1}{2}(x+3)^3 - 5)$$

$$\text{oftewel } y = 1\frac{1}{2}(x+3)^3 + 15$$

b Het punt van symmetrie is $(-3, 15)$.

25

a

$$y = 0,3x^4$$

↓ translatie $(-5, 6)$

$$y = 0,3(x+5)^4 + 6$$

↓ verm. x -as, -4

$$y = -4(0,3(x+5)^4 + 6)$$

$$\text{oftewel } y = -1,2(x+5)^4 - 24$$

De top is $(-5, -24)$.

b

$$y = 0,3x^4$$

↓ verm. x -as, -4

$$y = -1,2x^4$$

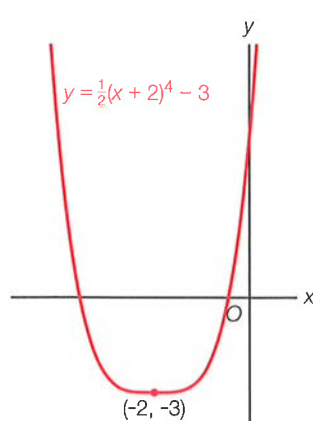
↓ translatie $(-5, 6)$

$$y = -1,2(x+5)^4 + 6$$

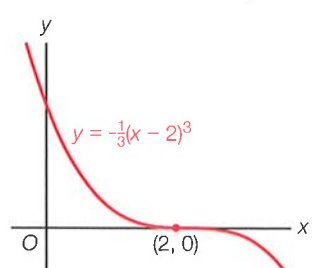
De top is $(-5, 6)$.

26

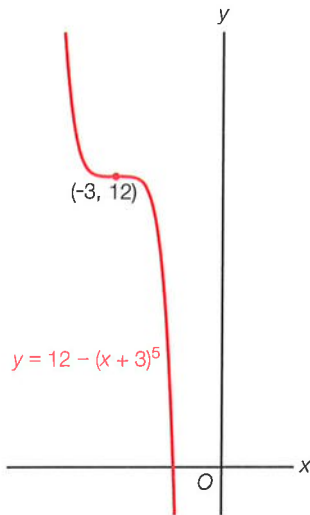
a



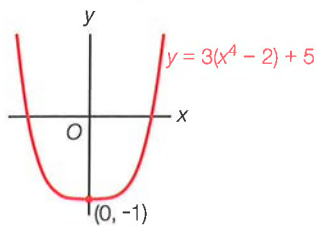
b



c



d $y = 3(x^4 - 2) + 5 = 3x^4 - 6 + 5 = 3x^4 - 1$



27

a $f(x) = \frac{1}{2}(x - 3)^5 + 7$

↓ translatie (1, 2)

$$y = \frac{1}{2}(x - 1 - 3)^5 + 7 + 2$$

oftewel $y = \frac{1}{2}(x - 4)^5 + 9$

↓ verm. x-as, -1

$$y = -\frac{1}{2}(x - 4)^5 - 9$$

Het punt van symmetrie is (4, -9).

b $g(x) = -2\frac{1}{2}(x + 4)^6 - 1$

↓ verm. x-as, -4

$$y = 10(x + 4)^6 + 4$$

↓ translatie (-5, -4)

$$h(x) = 10(x + 5 + 4)^6 + 4 - 4$$

oftewel $h(x) = 10(x + 9)^6$

min. is $h(-9) = 0$

28

a Voor f geldt $y = (x - 5)^3 + 5$, dus voor f^{inv} geldt $x = (y - 5)^3 + 5$.

$$x = (y - 5)^3 + 5 \text{ geeft } (y - 5)^3 + 5 = x$$

$$(y - 5)^3 = x - 5$$

$$y - 5 = \sqrt[3]{x - 5}$$

$$y = \sqrt[3]{x - 5} + 5$$

Dus $g(x) = \sqrt[3]{x - 5} + 5$ is de inverse van f .

b $f(x) = g(x)$ geeft $(x - 5)^3 + 5 = \sqrt[3]{x - 5} + 5$

$$(x - 5)^3 = \sqrt[3]{x - 5}$$

$$(x - 5)^9 = x - 5$$

$$(x - 5)^8 \cdot (x - 5) = x - 5$$

$$x - 5 = 0 \vee (x - 5)^8 = 1$$

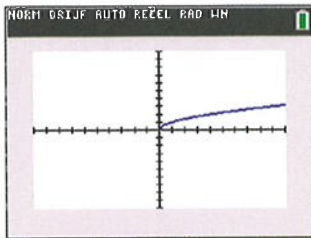
$$x = 5 \vee x - 5 = 1 \vee x - 5 = -1$$

$$x = 5 \vee x = 6 \vee x = 4$$

De snijpunten zijn (4, 4), (5, 5) en (6, 6).

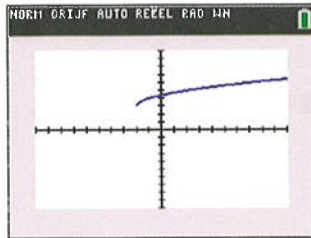
29

a



Het domein is $[0, \rightarrow)$ en het bereik is $[0, \rightarrow)$.

b



De grafiek van $y = \sqrt{x+2} + 3$ ontstaat uit de grafiek van $y = \sqrt{x}$ bij de translatie $(-2, 3)$.

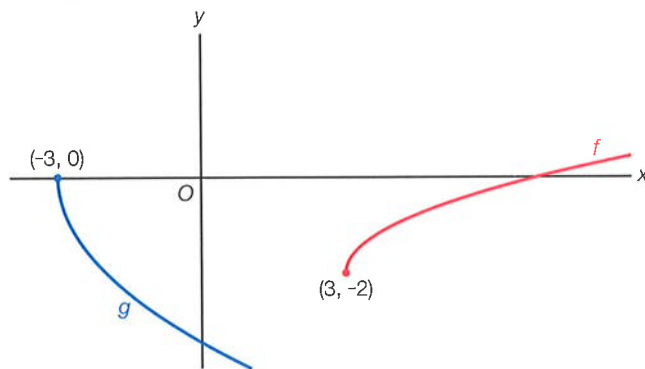
Bladzijde 23

30

a

De grafiek van $f(x) = \sqrt{x-3} - 2$ ontstaat uit die van $y = \sqrt{x}$ bij de translatie $(3, -2)$.
De grafiek van $g(x) = -2\sqrt{x+3}$ ontstaat uit die van $y = \sqrt{x}$ bij de vermenigvuldiging ten opzichte van de x -as met -2 en de translatie $(-3, 0)$.

b



c $D_f = [3, \rightarrow)$, $B_f = [-2, \rightarrow)$, $D_g = [-3, \rightarrow)$ en $B_g = \langle \leftarrow, 0]$

Bladzijde 24

31

a

$$y = \sqrt{x}$$

↓ verm. x -as, 2

$$y = 2\sqrt{x}$$

↓ translatie $(0, -3)$

$$f(x) = 2\sqrt{x} - 3$$

$$y = \sqrt{x}$$

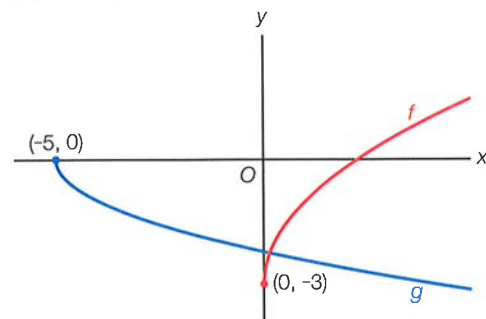
↓ verm. x -as, -1

$$y = -\sqrt{x}$$

↓ translatie $(-5, 0)$

$$g(x) = -\sqrt{x+5}$$

b



c $D_f = [0, \rightarrow)$, $B_f = [-3, \rightarrow)$, $D_g = [-5, \rightarrow)$ en $B_g = \langle \leftarrow, 0]$

- 32** a randpunt $(-5, 3)$, $D_f = [-5, \rightarrow)$ en $B_f = [3, \rightarrow)$
 b randpunt $(-3, -7)$, $D_g = [-3, \rightarrow)$ en $B_g = [-7, \rightarrow)$
 c randpunt $(-1, 0)$, $D_h = [-1, \rightarrow)$ en $B_h = \langle \leftarrow, 0]$
 d randpunt $(0, 1)$, $D_k = [0, \rightarrow)$ en $B_k = [1, \rightarrow)$
 e randpunt $(1, -1)$, $D_l = [1, \rightarrow)$ en $B_l = \langle \leftarrow, -1]$
 f randpunt $(0, -3)$, $D_m = [0, \rightarrow)$ en $B_m = [-3, \rightarrow)$

- 33** a Voor het domein van $f(x) = 5 - \sqrt{2x - 6}$ moet gelden $2x - 6 \geq 0$

$$2x \geq 6$$

$$x \geq 3$$

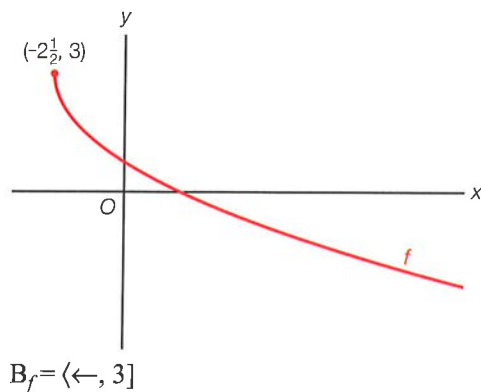
Dus het domein is $D_f = [3, \rightarrow)$.

- b De uitkomst van een wortel is minstens 0, dus de uitkomst van $f(x)$ is hoogstens 5.
 Dus het bereik is $B_f = \langle \leftarrow, 5]$.

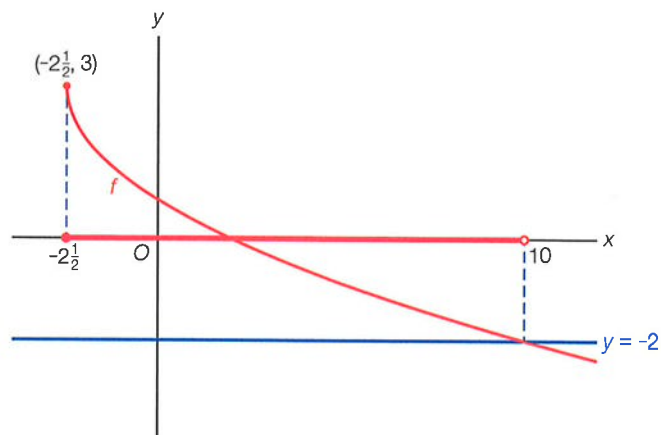
Bladzijde 26

- 34** a $8 - 4x \geq 0$
 $-4x \geq -8$
 $x \leq 2$
 Dus $D_f = \langle \leftarrow, 2]$, $B_f = [3, \rightarrow)$ en randpunt $(2, 3)$.
 b $4x - 8 \geq 0$
 $4x \geq 8$
 $x \geq 2$
 Dus $D_g = [2, \rightarrow)$, $B_g = [3, \rightarrow)$ en randpunt $(2, 3)$.
 c $2x + 6 \geq 0$
 $2x \geq -6$
 $x \geq -3$
 Dus $D_h = [-3, \rightarrow)$, $B_h = \langle \leftarrow, 5]$ en randpunt $(-3, 5)$.
 d $x \geq 0$
 Dus $D_k = [0, \rightarrow)$, $B_k = \langle \leftarrow, 3]$ en randpunt $(0, 3)$.

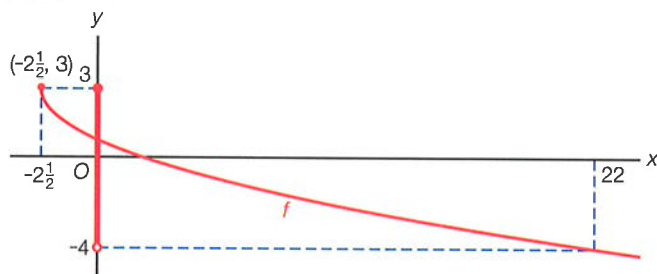
- 35** a $2x + 5 \geq 0$
 $2x \geq -5$
 $x \geq -2\frac{1}{2}$
 $D_f = [-2\frac{1}{2}, \rightarrow)$
 randpunt $(-2\frac{1}{2}, 3)$



b $f(x) = -2$ geeft $3 - \sqrt{2x+5} = -2$
 $\sqrt{2x+5} = 5$
 $2x+5 = 25$
 $2x = 20$
 $x = 10$

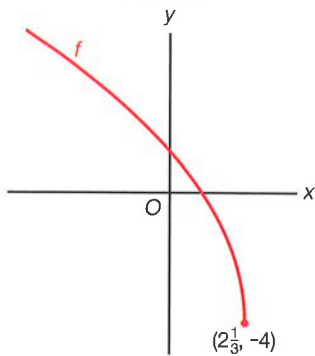


$f(x) > -2$ geeft $-2\frac{1}{2} \leq x < 10$
c $f(22) = -4$



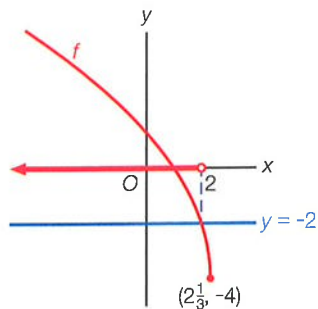
$x < 22$ geeft $-4 < f(x) \leq 3$

36 a $7 - 3x \geq 0$
 $-3x \geq -7$
 $x \leq 2\frac{1}{3}$ dus $D_f = \langle \leftarrow, 2\frac{1}{3} \rangle$
 randpunt $(2\frac{1}{3}, -4)$



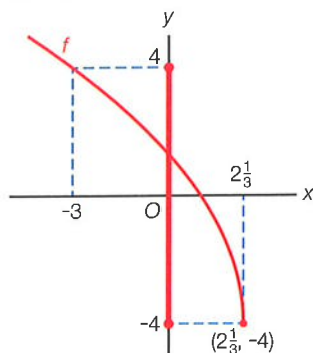
$B_f = [-4, \rightarrow)$

b $f(x) = -2$ geeft $2\sqrt{7-3x} - 4 = -2$
 $2\sqrt{7-3x} = 2$
 $\sqrt{7-3x} = 1$
 $7-3x = 1$
 $-3x = -6$
 $x = 2$



$f(x) > -2$ geeft $x < 2$

c $f(-3) = 4$



$x \geq -3$ geeft $-4 \leq f(x) \leq 4$

37 Stel $f(x) = 2 + \sqrt{7-2x}$ en $g(x) = x$.

$7-2x \geq 0$

$-2x \geq -7$

$x \leq 3\frac{1}{2}$

Dus $D_f = \langle \leftarrow, 3\frac{1}{2} \rangle$ en het randpunt van de grafiek van f is $(3\frac{1}{2}, 2)$.

$f(x) = g(x)$ geeft $2 + \sqrt{7-2x} = x$

$\sqrt{7-2x} = x-2$

kwadrateren geeft

$7-2x = x^2 - 4x + 4$

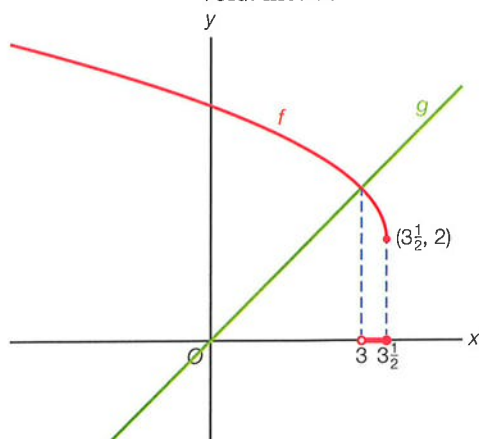
$-x^2 + 2x + 3 = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1 \vee x = 3$

vold. niet vold.



$2 + \sqrt{7-2x} < x$ geeft $3 < x \leq 3\frac{1}{2}$

38 Er geldt $D_f = [-8, \rightarrow)$, $B_f = \langle \leftarrow, 3 \rangle$ en $g = f^{\text{inv}}$, dus $D_g = B_f = \langle \leftarrow, 3 \rangle$ en $B_g = D_f = [-8, \rightarrow)$.

39 $5 + ax = 0$

$$ax = -5$$

$$x = -\frac{5}{a}$$

Dus het randpunt van de grafiek van f is $\left(-\frac{5}{a}, 4\right)$.

$$\left(-\frac{5}{a}, 4\right) \text{ op de lijn } y = 2x - 1 \text{ geeft } 2 \cdot -\frac{5}{a} - 1 = 4$$

$$-\frac{10}{a} = 5$$

$$a = -2$$

40 $(5, 3)$ op de grafiek van $f(x) = a\sqrt{x+b}$ geeft $a\sqrt{5+b} = 3$

$$a = \frac{3}{\sqrt{b+5}}$$

$(13, 9)$ op de grafiek van $f(x) = a\sqrt{x+b}$ geeft $a\sqrt{13+b} = 9$

$$a = \frac{9}{\sqrt{b+13}}$$

$$a = \frac{3}{\sqrt{b+5}} \text{ en } a = \frac{9}{\sqrt{b+13}} \text{ geeft } \frac{3}{\sqrt{b+5}} = \frac{9}{\sqrt{b+13}}$$

$$3\sqrt{b+13} = 9\sqrt{b+5}$$

$$\sqrt{b+13} = 3\sqrt{b+5}$$

kwadrateren geeft

$$b+13 = 9(b+5)$$

$$b+13 = 9b+45$$

$$-8b = 32$$

$$b = -4 \text{ vold.}$$

$$b = -4 \text{ en } a = \frac{3}{\sqrt{b+5}} \text{ geeft } a = \frac{3}{\sqrt{-4+5}} = 3$$

Dus $a = 3$ en $b = -4$.

41 a $y = 2\sqrt{x}$ geeft $2\sqrt{x} = y$

kwadrateren geeft

$$4x = y^2$$

$$x = \frac{1}{4}y^2$$

b $y = \sqrt{x-2}$ geeft $\sqrt{x-2} = y$

kwadrateren geeft

$$x-2 = y^2$$

$$x = y^2 + 2$$

Uit $y = \sqrt{x-2}$ volgt $x = y^2 + 2$.

c $y = 2\sqrt{x-2}$ geeft $2\sqrt{x-2} = y$

kwadrateren geeft

$$4(x-2) = y^2$$

$$x-2 = \frac{1}{4}y^2$$

$$x = \frac{1}{4}y^2 + 2$$

Uit $y = 2\sqrt{x-2}$ volgt $x = \frac{1}{4}y^2 + 2$.

Bladzijde 27

- 42 a** $F = 3\sqrt{2t-1}$ geeft $3\sqrt{2t-1} = F$
 kwadrateren geeft
 $9(2t-1) = F^2$
 $2t-1 = \frac{1}{9}F^2$
 $2t = \frac{1}{9}F^2 + 1$
 $t = \frac{1}{18}F^2 + \frac{1}{2}$
- b** $A = 5 + \sqrt{4-3B}$ geeft $5 + \sqrt{4-3B} = A$
 $\sqrt{4-3B} = A-5$
 kwadrateren geeft
 $4-3B = (A-5)^2$
 $4-3B = A^2 - 10A + 25$
 $-3B = A^2 - 10A + 21$
 $B = -\frac{1}{3}A^2 + 3\frac{1}{3}A - 7$
- c** $2x\sqrt{y} - 5 = 0$ geeft $2x\sqrt{y} = 5$
 kwadrateren geeft
 $4x^2y = 25$
 $y = \frac{25}{4x^2}$
- d** $R\sqrt{q} - \sqrt{R} = 6$ geeft $R\sqrt{q} = 6 + \sqrt{R}$
 kwadrateren geeft
 $R^2q = (6 + \sqrt{R})^2$
 $R^2q = 36 + 12\sqrt{R} + R$
 $q = \frac{36 + 12\sqrt{R} + R}{R^2}$

Bladzijde 28

- 43 a** $T = -27,4$ geeft $v = 331\sqrt{1 - \frac{27,4}{273}} = 313,9...$
 $T = 38,6$ geeft $v = 331\sqrt{1 + \frac{38,6}{273}} = 353,6...$
 Het verschil is $353,6... - 313,9... \approx 40$ m/s.
- b** $v = 340$ geeft $331\sqrt{1 + \frac{T}{273}} = 340$
 $\sqrt{1 + \frac{T}{273}} = \frac{340}{331}$
 kwadrateren geeft
 $1 + \frac{T}{273} = \left(\frac{340}{331}\right)^2$
 $\frac{T}{273} = \left(\frac{340}{331}\right)^2 - 1$
 $T = 273 \cdot \left(\frac{340}{331}\right)^2 - 273 \approx 15$
- Bij een temperatuur van 15°C is de geluidssnelheid 340 m/s.
- c** $T_0 = 25$ en $h = 2,5$ geeft $T = 25 - 6,5 \cdot 2,5 = 8,75$
 $T = 8,75$ geeft $v = 331\sqrt{1 + \frac{8,75}{273}} = 336,2... \text{ m/s} = 1210,5... \text{ km/uur}$
 De snelheid is 1211 km/uur.

d $T = -1$ en $h = 2$ geeft $T_0 - 6,5 \cdot 2 = -1$

$$T_0 = 12$$

Dus $T = 12 - 6,5h$.

Dit geeft $v = 331 \sqrt{1 + \frac{12 - 6,5h}{273}} = 331 \sqrt{1 + \frac{12}{273} - \frac{6,5}{273}h} \approx 331 \sqrt{-0,0238h + 1,0440}$.

Dus $v = 331 \sqrt{-0,0238h + 1,0440}$.

e $v = 331 \sqrt{1 + \frac{T}{273}}$ geeft $331 \sqrt{1 + \frac{T}{273}} = v$

$$\sqrt{1 + \frac{T}{273}} = \frac{v}{331}$$

kwadrateren geeft

$$1 + \frac{T}{273} = \left(\frac{v}{331}\right)^2$$

$$\frac{T}{273} = \left(\frac{v}{331}\right)^2 - 1$$

$$T = 273 \cdot \left(\frac{v}{331}\right)^2 - 273$$

$$T = \frac{273}{109561}v^2 - 273$$

$T = T_0 - 6,5h$ en $T = \frac{273}{109561}v^2 - 273$ geeft $T_0 - 6,5h = \frac{273}{109561}v^2 - 273$

$$-6,5h = \frac{273}{109561}v^2 - T_0 - 273$$

$$h = -\frac{273}{109561 \cdot 6,5}v^2 + \frac{1}{6,5}T_0 + \frac{273}{6,5}$$

$$h \approx -0,0004v^2 + 0,1538T_0 + 42$$

Dus $h = -0,0004v^2 + 0,1538T_0 + 42$.

44 Voor f geldt $y = a + \sqrt{bx + c}$, dus voor f^{inv} geldt $x = a + \sqrt{by + c}$.

$x = a + \sqrt{by + c}$ geeft $a + \sqrt{by + c} = x$

$$\sqrt{by + c} = x - a$$

kwadrateren geeft

$$by + c = (x - a)^2$$

$$by + c = x^2 - 2ax + a^2$$

$$by = x^2 - 2ax + a^2 - c$$

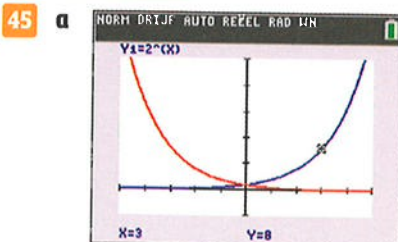
$$y = \frac{1}{b}x^2 - \frac{2a}{b}x + \frac{a^2 - c}{b}$$

$f^{\text{inv}}(x) = \frac{1}{2}x^2 - 3x$ geeft $\frac{1}{b} = \frac{1}{2} \wedge -\frac{2a}{b} = -3 \wedge \frac{a^2 - c}{b} = 0$

Dus $b = 2 \wedge -a = -3 \wedge a^2 - c = 0$ oftewel $a = 3$, $b = 2$ en $c = 9$.

5.3 Exponentiële functies

Bladzijde 30



De grafieken zijn elkaars gespiegelde in de y -as.

- b** $f(-10) = 2^{-10} \approx 9,77 \cdot 10^{-4}$
 $f(-20) = 2^{-20} \approx 9,54 \cdot 10^{-7}$
 $f(-100) = 2^{-100} \approx 7,89 \cdot 10^{-31}$
- c** Voor elke x is $2^x > 0$, dus er is geen origineel te vinden waarvan het beeld 0 is.
- d** $B_f = B_g = \langle 0, \rightarrow \rangle$

Bladzijde 32

- 46 a** $y = 3^x$
 $\downarrow$ translatie $(-2, -1)$
 $y = 3^{x+2} - 1$
 De asymptoot is de lijn $y = -1$.
- b** $y = 0,5^x$
 $\downarrow$ verm. x -as, 2
 $y = 2 \cdot 0,5^x$
 $\downarrow$ translatie $(0, 3)$
 $y = 2 \cdot 0,5^x + 3$
 De asymptoot is de lijn $y = 3$.
- c** $y = 2^x$
 $\downarrow$ verm. y -as, $\frac{1}{3}$
 $y = 2^{3x}$
 $\downarrow$ verm. x -as, 3
 $y = 3 \cdot 2^{3x}$
 $\downarrow$ translatie $(0, 4)$
 $y = 3 \cdot 2^{3x} + 4$
 De asymptoot is de lijn $y = 4$.
- d** $y = 0,8^x$
 $\downarrow$ verm. x -as, -1
 $y = -0,8^x$
 $\downarrow$ translatie $(-1, 10)$
 $y = 10 - 0,8^{x+1}$
 $\downarrow$ verm. y -as, 2,5
 $y = 10 - 0,8^{0,4x+1}$
 De asymptoot is de lijn $y = 10$.

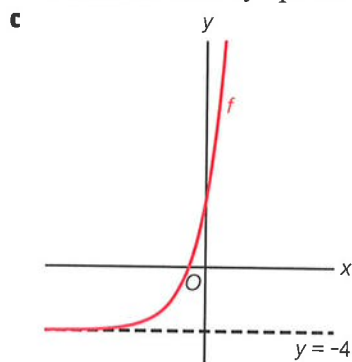
- 47 a** Het bereik is $\langle -6, \rightarrow \rangle$ en de asymptoot is de lijn $y = -6$.
- b** Het bereik is $\langle \leftarrow, 5 \rangle$ en de asymptoot is de lijn $y = 5$.
- c** Het bereik is $\langle \leftarrow, 1000 \rangle$ en de asymptoot is de lijn $y = 1000$.
- d** Het bereik is $\langle \leftarrow, 1000 \rangle$ en de asymptoot is de lijn $y = 1000$.

- 48 a** $y = 3^x$
 $\downarrow$ spiegelen in de x -as
 $y = -3^x$
 $\downarrow$ translatie $(0, -1)$
 $y = -3^x - 1$
- b** $y = 3^x$
 $\downarrow$ translatie $(2, 5)$
 $y = 3^{x-2} + 5$
 $\downarrow$ verm. y -as, $\frac{1}{4}$
 $y = 3^{4x-2} + 5$
- c** $y = 3^x$
 $\downarrow$ translatie $(4, -5)$
 $y = 3^{x-4} - 5$
 $\downarrow$ verm. x -as, 3
 $y = 3(3^{x-4} - 5)$
 oftewel $y = 3^{x-3} - 15$

- d** $y = 3^x$
 $\downarrow$ verm. x -as, 3
 $y = 3 \cdot 3^x$
 $\downarrow$ translatie (4, -5)
 $y = 3 \cdot 3^{x-4} - 5$
 oftewel $y = 3^{x-3} - 5$

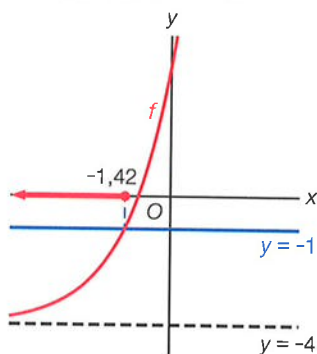
- 49 a** $y = 2^x$
 $\downarrow$ translatie (-3, -4)
 $f(x) = 2^{x+3} - 4$

- b** $B_f = \langle -4, \rightarrow \rangle$
 De horizontale asymptoot is de lijn $y = -4$.



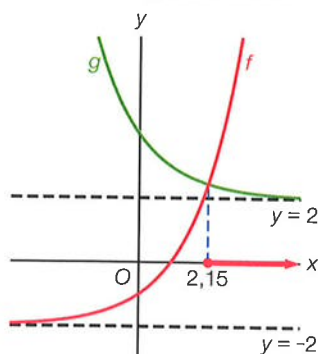
- d** $f(3) = 60$
 $x \leq 3$ geeft $-4 < f(x) \leq 60$

- e** Voer in $y_2 = -1$.
 De optie snijpunt geeft $x \approx -1,42$.



$f(x) \leq -1$ geeft $x \leq -1,42$

- 50 a** Voer in $y_1 = 2^x - 2$ en $y_2 = (\frac{1}{2})^{x-1} + 2$.
 De optie snijpunt geeft $x \approx 2,15$.



$f(x) \geq g(x)$ geeft $x \geq 2,15$

- b** $B_f = \langle -2, \rightarrow \rangle$, dus $f(x) = p$ heeft geen oplossingen voor $p \leq -2$.

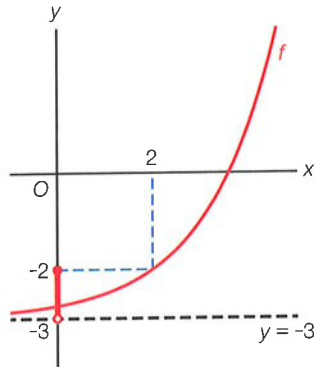
Bladzijde 33

- 51 a** $y = 2^x$
 $\downarrow$ translatie $(2, -3)$
 $f(x) = 2^{x-2} - 3$
 $B_f = \langle -3, \rightarrow \rangle$
- $y = 0,5^x$
 $\downarrow$ verm. x-as 4
 $y = 4 \cdot 0,5^x$
 $\downarrow$ translatie $(3, -1)$
 $g(x) = 4 \cdot 0,5^{x-3} - 1$
 $B_g = \langle -1, \rightarrow \rangle$

$B_f = \langle -3, \rightarrow \rangle$ en $B_g = \langle -1, \rightarrow \rangle$, dus $f(x) = p$ heeft één oplossing voor $p > -3$ en $g(x) = p$ heeft geen oplossingen voor $p \leq -1$.

Dus voor $-3 < p \leq -1$ heeft $f(x) = p$ één oplossing en $g(x) = p$ geen oplossingen.

- b** $f(2) = -2$



$x \leq 2$ geeft $-3 < f(x) \leq -2$

- c** $f(1) = -2\frac{1}{2}$ en $g(1) = 15$, dus $A(1, -2\frac{1}{2})$ en $B(1, 15)$.
 $AB = y_B - y_A = 15 - (-2\frac{1}{2}) = 17\frac{1}{2}$
- d** Voer in $y_1 = 2^{x-2} - 3$, $y_2 = 4 \cdot 0,5^{x-3} - 1$ en $y_3 = 5$.
 De optie snijpunt met y_1 en y_3 geeft $x = 5$, dus $P(5, 5)$.
 De optie snijpunt met y_2 en y_3 geeft $x = 2,415\dots$, dus $Q(2,415\dots; 5)$.
 $PQ = x_P - x_Q = 5 - 2,415\dots \approx 2,585$

- 52 a** $8 \cdot 4^x = 2^3 \cdot (2^2)^x = 2^3 \cdot 2^{2x} = 2^{2x+3}$
- b** Bij de herleiding van $(2^2)^x$ tot 2^{2x} is de regel $(a^p)^q = a^{pq}$ gebruikt.
 Bij de herleiding van $2^3 \cdot 2^{2x}$ tot 2^{2x+3} is de regel $a^p \cdot a^q = a^{p+q}$ gebruikt.

- 53 a** $y = 15 \cdot 2^{3x+2} = 15 \cdot 2^{3x} \cdot 2^2 = 15 \cdot (2^3)^x \cdot 4 = 60 \cdot 8^x$
 Dus $y = 60 \cdot 8^x$.
- b** $y = 50 \cdot 2^{3x-1} = 50 \cdot 2^{3x} \cdot 2^{-1} = 50 \cdot (2^3)^x \cdot \frac{1}{2} = 25 \cdot 8^x$
 Dus $y = 25 \cdot 8^x$.
- c** $y = 260 \cdot 4^{1\frac{1}{2}x-1} = 260 \cdot (2^2)^{1\frac{1}{2}x-1} = 260 \cdot 2^{3x-2} = 260 \cdot (2^3)^x \cdot 2^{-2} = 260 \cdot 8^x \cdot \frac{1}{4} = 65 \cdot 8^x$
 Dus $y = 65 \cdot 8^x$.
- d** $y = 8 \cdot 4^{-2x-1} = 8 \cdot 4^{-2x} \cdot 4^{-1} = 8 \cdot (4^{-2})^x \cdot \frac{1}{4} = 2 \cdot (\frac{1}{16})^x$
 Dus $y = 2 \cdot (\frac{1}{16})^x$.

Bladzijde 34

- 54 a** $y = 2^x \cdot 2^{2x-3} = 2^x \cdot 2^{2x} \cdot 2^{-3} = 2^{3x} \cdot \frac{1}{8} = \frac{1}{8} \cdot (2^3)^x = \frac{1}{8} \cdot 8^x$
 Dus $y = \frac{1}{8} \cdot 8^x$.
- b** $y = 63 \cdot 3^{\frac{1}{2}x-2} = 63 \cdot 3^{\frac{1}{2}x} \cdot 3^{-2} = 63 \cdot (3^{\frac{1}{2}})^x \cdot \frac{1}{9} = 7 \cdot (\sqrt{3})^x$
 Dus $y = 7 \cdot (\sqrt{3})^x$.
- c** $y = 50\,000 \cdot 100^{-x-2\frac{1}{2}} = 50\,000 \cdot 100^{-x} \cdot 100^{-2\frac{1}{2}} = 50\,000 \cdot (100^{-1})^x \cdot 10^{-5} = \frac{1}{2} \cdot (\frac{1}{100})^x$
 Dus $y = \frac{1}{2} \cdot (\frac{1}{100})^x$.
- d** $y = \frac{3}{4^{2x-1}} = 3 \cdot 4^{-2x+1} = 3 \cdot 4^{-2x} \cdot 4^1 = 3 \cdot (4^{-2})^x \cdot 4 = 12 \cdot (\frac{1}{16})^x$
 Dus $y = 12 \cdot (\frac{1}{16})^x$.

55 $f(x) = 5 - 2^{\frac{1}{2}x+4}$
 $\downarrow$ translatie (12, -8)
 $h(x) = 5 - 2^{\frac{1}{2}(x-12)+4} - 8 = -3 - 2^{\frac{1}{2}x-6+4} = -3 - 2^{\frac{1}{2}x-2} = -3 - 2^{\frac{1}{2}x} \cdot 2^{-2} = -3 - (2^{\frac{1}{2}})^x \cdot \frac{1}{4} = -3 - \frac{1}{4} \cdot (\sqrt{2})^x$
 Dus $a = -3$, $b = -\frac{1}{4}$ en $g = \sqrt{2}$.

56 $f(x) = 2^{x-3}$
 $\downarrow$ translatie (-5, 0)
 $g(x) = 2^{x+5-3} = 2^{x-3} \cdot 2^5 = 32 \cdot 2^{x-3}$
 De vermenigvuldiging ten opzichte van de x -as met 32 toepassen op de grafiek van f geeft de grafiek van g .
 Dus $a = 32$.

57 $4\sqrt{2} = 2^2 \cdot 2^{\frac{1}{2}} = 2^{2\frac{1}{2}}$
 $2^{x-1} = 4\sqrt{2}$
 $2^{x-1} = 2^{2\frac{1}{2}}$
 $x-1 = 2\frac{1}{2}$
 $x = 3\frac{1}{2}$

Bladzijde 35

58 a $2^{x+1} = 64$
 $2^{x+1} = 2^6$
 $x+1 = 6$
 $x = 5$
b $2^{x-3} = \frac{1}{8}$
 $2^{x-3} = 2^{-3}$
 $x-3 = -3$
 $x = 0$
c $3^{2x} = 3$
 $3^{2x} = 3^1$
 $2x = 1$
 $x = \frac{1}{2}$

d $(\frac{1}{2})^x = 1$
 $(\frac{1}{2})^x = (\frac{1}{2})^0$
 $x = 0$
e $2^x = \frac{1}{4}\sqrt{2}$
 $2^x = 2^{-2} \cdot 2^{\frac{1}{2}}$
 $2^x = 2^{-1\frac{1}{2}}$
 $x = -1\frac{1}{2}$
f $5^{x+6} = (\frac{1}{5})^x$
 $5^{x+6} = (5^{-1})^x$
 $5^{x+6} = 5^{-x}$
 $x+6 = -x$
 $2x = -6$
 $x = -3$

59 a $3^{2x+1} = 27\sqrt{3}$
 $3^{2x+1} = 3^3 \cdot 3^{\frac{1}{2}}$
 $3^{2x+1} = 3^{3\frac{1}{2}}$
 $2x+1 = 3\frac{1}{2}$
 $2x = 2\frac{1}{2}$
 $x = 1\frac{1}{4}$
b $10^{2x+1} = 0,01$
 $10^{2x+1} = 10^{-2}$
 $2x+1 = -2$
 $2x = -3$
 $x = -1\frac{1}{2}$
c $3^x - 2 = 25$
 $3^x = 27$
 $3^x = 3^3$
 $x = 3$

d $5 \cdot 2^x = 80$
 $2^x = 16$
 $2^x = 2^4$
 $x = 4$
e $10 \cdot 3^x = 270$
 $3^x = 27$
 $3^x = 3^3$
 $x = 3$
f $3 \cdot 8^{2-x} = 48$
 $8^{2-x} = 16$
 $(2^3)^{2-x} = 2^4$
 $2^{6-3x} = 2^4$
 $6-3x = 4$
 $-3x = -2$
 $x = \frac{2}{3}$

60 a $3 \cdot 2^x + 4 = 28$

$$3 \cdot 2^x = 24$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

b $5^{2x-6} = 0,04$

$$5^{2x-6} = \frac{1}{25}$$

$$5^{2x-6} = 5^{-2}$$

$$2x - 6 = -2$$

$$2x = 4$$

$$x = 2$$

c $3 \cdot 7^{3x+1} = 147$

$$7^{3x+1} = 49$$

$$7^{3x+1} = 7^2$$

$$3x + 1 = 2$$

$$3x = 1$$

$$x = \frac{1}{3}$$

d $32^{x-2} = \frac{1}{16}$

$$(2^5)^{x-2} = 2^{-4}$$

$$2^{5x-10} = 2^{-4}$$

$$5x - 10 = -4$$

$$5x = 6$$

$$x = 1\frac{1}{5}$$

e $5 \cdot 4^{x-1} = 2\frac{1}{2}$

$$4^{x-1} = \frac{1}{2}$$

$$(2^2)^{x-1} = 2^{-1}$$

$$2^{2x-2} = 2^{-1}$$

$$2x - 2 = -1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

f $8 \cdot 2^x = 4^{x+1}$

$$2^3 \cdot 2^x = (2^2)^{x+1}$$

$$2^{3+x} = 2^{2x+2}$$

$$3 + x = 2x + 2$$

$$-x = -1$$

$$x = 1$$

61 $f(x) = 3^{x+2} - 5$

↓ translatie (3, 7)

$$y = 3^{x-1} + 2$$

↓ verm. y-as, b

$$g(x) = 3^{\frac{1}{b}x-1} + 2$$

$$g(x) = 3^{\frac{1}{b}x-1} + 2 \left\{ \begin{array}{l} 3^{\frac{-15}{b}-1} + 2 = 83 \\ 3^{\frac{-15}{b}-1} = 81 \end{array} \right.$$

$$\text{door } A(-15, 83) \left\{ \begin{array}{l} 3^{\frac{-15}{b}-1} = 3^4 \end{array} \right.$$

$$3^{\frac{-15}{b}-1} = 3^4$$

$$\frac{-15}{b} - 1 = 4$$

$$\frac{-15}{b} = 5$$

$$b = -3$$

62 a $2^{x+1} = 2^x \cdot 2^1 = 2 \cdot 2^x$, dus uit $2^{x+1} + 2^x = 48$ volgt $2 \cdot 2^x + 2^x = 48$.

b $2 \cdot 2^x + 2^x = 2 \cdot 2^x + 1 \cdot 2^x = 3 \cdot 2^x$, dus uit $2 \cdot 2^x + 2^x = 48$ volgt $3 \cdot 2^x = 48$.

$$3 \cdot 2^x = 48$$

$$2^x = 16$$

$$2^x = 2^4$$

$$x = 4$$

63 a $3^{x+2} + 3^x = 810$

$$3^x \cdot 3^2 + 3^x = 810$$

$$9 \cdot 3^x + 3^x = 810$$

$$10 \cdot 3^x = 810$$

$$3^x = 81$$

$$3^x = 3^4$$

$$x = 4$$

b $2^{x-1} + 2^{x+1} = 10$

$$2^x \cdot 2^{-1} + 2^x \cdot 2^1 = 10$$

$$\frac{1}{2} \cdot 2^x + 2 \cdot 2^x = 10$$

$$2\frac{1}{2} \cdot 2^x = 10$$

$$2^x = 4$$

$$2^x = 2^2$$

$$x = 2$$

c $2^{x+3} - 2^x = \frac{7}{8}$

$$2^x \cdot 2^3 - 2^x = \frac{7}{8}$$

$$8 \cdot 2^x - 2^x = \frac{7}{8}$$

$$7 \cdot 2^x = \frac{7}{8}$$

$$2^x = \frac{1}{8}$$

$$2^x = 2^{-3}$$

$$x = -3$$

d $3^{x+2} = 24 + 3^x$

$$3^x \cdot 3^2 = 24 + 3^x$$

$$9 \cdot 3^x - 3^x = 24$$

$$8 \cdot 3^x = 24$$

$$3^x = 3$$

$$3^x = 3^1$$

$$x = 1$$

$$\begin{aligned}
 \text{e} \quad 3^x - 3^{x-1} &= 2\sqrt{3} \\
 3^x - 3^x \cdot 3^{-1} &= 2\sqrt{3} \\
 3^x - \frac{1}{3} \cdot 3^x &= 2\sqrt{3} \\
 \frac{2}{3} \cdot 3^x &= 2\sqrt{3} \\
 3^x &= 3\sqrt{3} \\
 3^x &= 3^{1\frac{1}{2}} \\
 x &= 1\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{f} \quad 5^{x-1} + 5^{x-2} &= 6\sqrt{5} \\
 5^x \cdot 5^{-1} + 5^x \cdot 5^{-2} &= 6\sqrt{5} \\
 \frac{1}{5} \cdot 5^x + \frac{1}{25} \cdot 5^x &= 6\sqrt{5} \\
 \frac{6}{25} \cdot 5^x &= 6\sqrt{5} \\
 5^x &= 25\sqrt{5} \\
 5^x &= 5^{2\frac{1}{2}} \\
 x &= 2\frac{1}{2}
 \end{aligned}$$

Bladzijde 36

$$\begin{aligned}
 64 \quad \text{a} \quad 3^{x+1} &= 9^{x+2} \\
 3^{x+1} &= (3^2)^{x+2} \\
 3^{x+1} &= 3^{2x+4} \\
 x+1 &= 2x+4 \\
 -x &= 3 \\
 x &= -3
 \end{aligned}$$

$$\begin{aligned}
 \text{b} \quad 3^{x+1} - 3^{x-1} &= 8\sqrt{3} \\
 3^x \cdot 3^1 - 3^x \cdot 3^{-1} &= 8\sqrt{3} \\
 3 \cdot 3^x - \frac{1}{3} \cdot 3^x &= 8\sqrt{3} \\
 2\frac{2}{3} \cdot 3^x &= 8\sqrt{3} \\
 3^x &= 3\sqrt{3} \\
 3^x &= 3^{1\frac{1}{2}} \\
 x &= 1\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{c} \quad 3^{x^2} &= \left(\frac{1}{3}\right)^{x-6} \\
 3^{x^2} &= (3^{-1})^{x-6} \\
 3^{x^2} &= 3^{-x+6} \\
 x^2 &= -x+6 \\
 x^2 + x - 6 &= 0 \\
 (x-2)(x+3) &= 0 \\
 x &= 2 \vee x = -3
 \end{aligned}$$

$$\begin{aligned}
 \text{d} \quad 5^{x^2+5} &= 125^{x+1} \\
 5^{x^2+5} &= (5^3)^{x+1} \\
 5^{x^2+5} &= 5^{3x+3} \\
 x^2+5 &= 3x+3 \\
 x^2-3x+2 &= 0 \\
 (x-1)(x-2) &= 0 \\
 x &= 1 \vee x = 2 \\
 \text{e} \quad 2^{x+2} - \left(\frac{1}{2}\right)^{-x+1} &= 28 \\
 2^x \cdot 2^2 - (2^{-1})^{-x+1} &= 28 \\
 4 \cdot 2^x - 2^{x-1} &= 28 \\
 4 \cdot 2^x - 2^x \cdot 2^{-1} &= 28 \\
 4 \cdot 2^x - \frac{1}{2} \cdot 2^x &= 28 \\
 3\frac{1}{2} \cdot 2^x &= 28 \\
 2^x &= 8 \\
 2^x &= 2^3 \\
 x &= 3 \\
 \text{f} \quad 4^{x^2+1} &= 8^{x^2-1} \\
 (2^2)^{x^2+1} &= (2^3)^{x^2-1} \\
 2^{2x^2+2} &= 2^{3x^2-3} \\
 2x^2+2 &= 3x^2-3 \\
 -x^2 &= -5 \\
 x^2 &= 5 \\
 x &= \sqrt{5} \vee x = -\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 65 \quad f(x) &= 2^x \\
 \downarrow \text{translatie } (3, 8) \\
 y &= 2^{x-3} + 8 \\
 \downarrow \text{verm. } x\text{-as, } 4 \\
 g(x) &= 4 \cdot 2^{x-3} + 32 \\
 f(x) = g(x) \text{ geeft } 2^x &= 4 \cdot 2^{x-3} + 32 \\
 2^x &= 4 \cdot 2^x \cdot 2^{-3} + 32 \\
 2^x &= \frac{1}{2} \cdot 2^x + 32 \\
 \frac{1}{2} \cdot 2^x &= 32 \\
 2^x &= 64 \\
 2^x &= 2^6 \\
 x &= 6
 \end{aligned}$$

Het snijpunt van de grafieken van f en g is het punt $(6, 64)$.

$$\begin{aligned}
 66 \quad \text{a} \quad f(x) = g(x) \text{ geeft } 3^{x+1} - 4 &= 6 - 3^{x-1} \\
 3^x \cdot 3 - 4 &= 6 - 3^x \cdot 3^{-1} \\
 3 \cdot 3^x - 4 &= 6 - \frac{1}{3} \cdot 3^x \\
 3\frac{1}{3} \cdot 3^x &= 10 \\
 3^x &= 3 \\
 x &= 1 \\
 f(x) \leq g(x) \text{ geeft } x &\leq 1
 \end{aligned}$$

- b $f(2^{\frac{1}{2}}) = 3^{3^{\frac{1}{2}}} - 4 = 3^3 \cdot 3^{\frac{1}{2}} - 4 = 27\sqrt{3} - 4$, dus $A(2^{\frac{1}{2}}, 27\sqrt{3} - 4)$
 $g(2^{\frac{1}{2}}) = 6 - 3^{1^{\frac{1}{2}}} = 6 - 3 \cdot 3^{\frac{1}{2}} = 6 - 3\sqrt{3}$, dus $B(2^{\frac{1}{2}}, 6 - 3\sqrt{3})$
 $AB = y_A - y_B = 27\sqrt{3} - 4 - (6 - 3\sqrt{3}) = 30\sqrt{3} - 10$
- c $f(x) - g(x) = 80$ geeft $3^{x+1} - 4 - (6 - 3^{x-1}) = 80$
 $3^x \cdot 3 - 4 - 6 + 3^x \cdot 3^{-1} = 80$
 $3 \cdot 3^x - 10 + \frac{1}{3} \cdot 3^x = 80$
 $3^{\frac{1}{3}} \cdot 3^x = 90$
 $3^x = 27$
 $3^x = 3^3$
 $x = 3$
- d $g(x) - f(x) = 6 - 3^{x-1} - (3^{x+1} - 4) = 6 - 3^x \cdot 3^{-1} - 3^x \cdot 3 + 4 = 6 - \frac{1}{3} \cdot 3^x - 3 \cdot 3^x + 4 = 10 - 3^{\frac{1}{3}} \cdot 3^x$
 Het bereik van $g(x) - f(x)$ is $\langle \leftarrow, 10 \rangle$.
 Dus de vergelijking $g(x) - f(x) = p$ heeft geen oplossingen voor $p \geq 10$.

5.4 Logaritmen

Bladzijde 38

- 67 a $2^3 = 8$
 b $2^{-2} = \frac{1}{4}$
 c $2^{\frac{1}{2}} = \sqrt{2}$
- d $3^2 = 9$
 e $3^{-3} = \frac{1}{27}$
 f $3^{\frac{1}{3}} = \sqrt[3]{3}$
- 68 a ${}^5\log(125) = {}^5\log(5^3) = 3$
 b ${}^{10}\log(0,1) = {}^{10}\log(10^{-1}) = -1$
 c ${}^2\log(4) = {}^2\log(2^2) = 2$
 d ${}^7\log(49) = {}^7\log(7^2) = 2$
 e ${}^2\log(\sqrt{2}) = {}^2\log(2^{\frac{1}{2}}) = \frac{1}{2}$
- f ${}^2\log(0,5) = {}^2\log(2^{-1}) = -1$
 g ${}^4\log(0,25) = {}^4\log(4^{-1}) = -1$
 h ${}^4\log(4) = {}^4\log(4^1) = 1$
 i ${}^4\log(1) = {}^4\log(4^0) = 0$

Bladzijde 39

- 69 a ${}^2\log(64\sqrt{2}) = {}^2\log(2^6 \cdot 2^{\frac{1}{2}}) = {}^2\log(2^{6\frac{1}{2}}) = 6\frac{1}{2}$
 b ${}^3\log(\frac{1}{9}\sqrt{3}) = {}^3\log(3^{-2} \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{-1\frac{1}{2}}) = -1\frac{1}{2}$
 c ${}^3\log(3^{21,5}) = 21,5$
 d ${}^5\log(\frac{1}{125}) = {}^5\log(5^{-3}) = -3$
 e ${}^{\frac{1}{3}}\log(\frac{1}{27}) = {}^{\frac{1}{3}}\log((\frac{1}{3})^3) = 3$
 f ${}^{\frac{1}{2}}\log(\frac{1}{4}) = {}^{\frac{1}{2}}\log((\frac{1}{2})^2) = 2$
 g ${}^2\log(\frac{1}{32} \cdot \sqrt[3]{2}) = {}^2\log(2^{-5} \cdot 2^{\frac{1}{3}}) = {}^2\log(2^{-4\frac{2}{3}}) = -4\frac{2}{3}$
 h ${}^5\log(1) = {}^5\log(5^0) = 0$
 i ${}^3\log(81 \cdot \sqrt[5]{27}) = {}^3\log(3^4 \cdot \sqrt[5]{3^3}) = {}^3\log(3^4 \cdot 3^{\frac{3}{5}}) = {}^3\log(3^{4\frac{3}{5}}) = 4\frac{3}{5}$
- 70 a ${}^3\log(9) = {}^3\log(3^2) = 2$
 Dus ${}^3\log(x) = 2$ geeft $x = 9$.
 b ${}^5\log(\frac{1}{25}) = {}^5\log(5^{-2}) = -2$
 Dus ${}^5\log(x) = -2$ geeft $x = \frac{1}{25}$.
- 71 a $x = {}^5\log(0,2) = {}^5\log(\frac{1}{5}) = {}^5\log(5^{-1}) = -1$
 b ${}^9\log(x) = \frac{1}{2}$
 $x = 9^{\frac{1}{2}} = \sqrt{9} = 3$
 c ${}^x\log(1000) = 3$
 $x^3 = 1000$
 $x = 10$

72 a ${}^3\log(x+2) = 2$
 $x+2 = 3^2$
 $x+2 = 9$
 $x = 7$

b $1 + {}^{\frac{1}{2}}\log(x) = 4$
 ${}^{\frac{1}{2}}\log(x) = 3$
 $x = (\frac{1}{2})^3$
 $x = \frac{1}{8}$

c ${}^3\log(2x+1) = 4$
 $2x+1 = 3^4$
 $2x+1 = 81$
 $2x = 80$
 $x = 40$

d $5 + {}^4\log(x) = 3$
 ${}^4\log(x) = -2$
 $x = 4^{-2}$
 $x = \frac{1}{16}$

e ${}^{\frac{1}{2}}\log(x-1) = 3$
 $x-1 = (\frac{1}{2})^3$
 $x-1 = \frac{1}{8}$
 $x = 1\frac{1}{8}$

f ${}^2\log(x^2-4) = 5$
 $x^2-4 = 2^5$
 $x^2-4 = 32$
 $x^2 = 36$
 $x = 6 \vee x = -6$

73 a $4 \cdot {}^3\log(x) = 2$
 ${}^3\log(x) = \frac{1}{2}$
 $x = 3^{\frac{1}{2}}$
 $x = \sqrt{3}$

b ${}^3\log(4x-1) = -2$
 $4x-1 = 3^{-2}$
 $4x-1 = \frac{1}{9}$
 $4x = 1\frac{1}{9}$
 $x = \frac{5}{18}$

c $3 + {}^2\log(x) = -1$
 ${}^2\log(x) = -4$
 $x = 2^{-4}$
 $x = \frac{1}{16}$

d ${}^5\log(3x+2) = 1$
 $3x+2 = 5^1$
 $3x+2 = 5$
 $3x = 3$
 $x = 1$

e ${}^3\log(0,4x-5) = 2$
 $0,4x-5 = 3^2$
 $0,4x-5 = 9$
 $0,4x = 14$
 $x = 35$

f $4 + 2 \cdot {}^2\log(x) = 7$
 $2 \cdot {}^2\log(x) = 3$
 ${}^2\log(x) = 1\frac{1}{2}$
 $x = 2^{1\frac{1}{2}}$
 $x = 2^1 \cdot 2^{\frac{1}{2}}$
 $x = 2\sqrt{2}$

74 Voer in $y_1 = 2^x$ en $y_2 = 30$.
 De optie snijpunt geeft $x \approx 4,91$.

Bladzijde 40

75 a $2^{x-1} = 15$
 $x-1 = {}^2\log(15)$
 $x = 1 + {}^2\log(15)$

b $1 + 2^x = 15$
 $2^x = 14$
 $x = {}^2\log(14)$

c $4 + 3^{x+1} = 25$
 $3^{x+1} = 21$
 $x+1 = {}^3\log(21)$
 $x = -1 + {}^3\log(21)$

d $14 - 2^{x+3} = 2$
 $-2^{x+3} = -12$
 $2^{x+3} = 12$
 $x+3 = {}^2\log(12)$
 $x = -3 + {}^2\log(12)$

e $7 + 4^{2x} = 12$
 $4^{2x} = 5$

$2x = {}^4\log(5)$
 $x = \frac{1}{2} \cdot {}^4\log(5)$

f $3 \cdot 5^{2x+1} = 60$
 $5^{2x+1} = 20$

$2x+1 = {}^5\log(20)$
 $2x = -1 + {}^5\log(20)$
 $x = -\frac{1}{2} + \frac{1}{2} \cdot {}^5\log(20)$

g $3^{x+2} + 3^x = 600$
 $3^x \cdot 3^2 + 3^x = 600$
 $9 \cdot 3^x + 3^x = 600$

$10 \cdot 3^x = 600$
 $3^x = 60$

$x = {}^3\log(60)$

h $2^{1+2x} = 4^x + 6$
 $2^1 \cdot 2^{2x} = 4^x + 6$
 $2 \cdot (2^2)^x = 4^x + 6$
 $2 \cdot 4^x = 4^x + 6$
 $4^x = 6$
 $x = {}^4\log(6)$

- 76** **a** $4^x = 2^{x+2} - 3$
 $(2^2)^x = 2^x \cdot 2^2 - 3$
 $(2^x)^2 = 4 \cdot 2^x - 3$
- b** 2^x bij de vergelijking $(2^x)^2 = 4 \cdot 2^x - 3$ vervangen door u geeft $u^2 = 4u - 3$.
- c** $u^2 = 4u - 3$
 $u^2 - 4u + 3 = 0$
 $(u - 1)(u - 3) = 0$
 $u = 1 \vee u = 3$
- d** $u = 1$ geeft $2^x = 1$ oftewel $x = 0$.
 $u = 3$ geeft $2^x = 3$ oftewel $x = {}^2\log(3)$.

- 77** **a** $9^x - 3^{x+1} = 4$
 $(3^2)^x - 3^x \cdot 3^1 = 4$
 $(3^x)^2 - 3 \cdot 3^x - 4 = 0$
Stel $3^x = u$.
 $u^2 - 3u - 4 = 0$
 $(u + 1)(u - 4) = 0$
 $u = -1 \vee u = 4$
 $3^x = -1 \vee 3^x = 4$
 $x = {}^3\log(4)$
- b** $4^x = 2^x + 42$
 $(2^2)^x = 2^x + 42$
 $(2^x)^2 - 2^x - 42 = 0$
Stel $2^x = u$.
 $u^2 - u - 42 = 0$
 $(u + 6)(u - 7) = 0$
 $u = -6 \vee u = 7$
 $2^x = -6 \vee 2^x = 7$
 $x = {}^2\log(7)$
- c** $2^x = 24 - 2^{2x-1}$
 $2^x = 24 - 2^{2x} \cdot 2^{-1}$
 $2^x = 24 - \frac{1}{2} \cdot (2^x)^2$
 $\frac{1}{2} \cdot (2^x)^2 + 2^x - 24 = 0$
 $(2^x)^2 + 2 \cdot 2^x - 48 = 0$
Stel $2^x = u$.
 $u^2 + 2u - 48 = 0$
 $(u - 6)(u + 8) = 0$
 $u = 6 \vee u = -8$
 $2^x = 6 \vee 2^x = -8$
 $x = {}^2\log(6)$
- d** $9^x = 5 \cdot 3^x + 6$
 $(3^2)^x = 5 \cdot 3^x + 6$
 $(3^x)^2 - 5 \cdot 3^x - 6 = 0$
Stel $3^x = u$.
 $u^2 - 5u - 6 = 0$
 $(u + 1)(u - 6) = 0$
 $u = -1 \vee u = 6$
 $3^x = -1 \vee 3^x = 6$
 $x = {}^3\log(6)$

- 78** **a** $5^{x-1} + 5^{2x-1} = 4$
 $5^x \cdot 5^{-1} + 5^{2x} \cdot 5^{-1} = 4$
 $\frac{1}{5} \cdot 5^x + \frac{1}{5} \cdot (5^x)^2 = 4$
 $5^x + (5^x)^2 = 20$
 $(5^x)^2 + 5^x - 20 = 0$
Stel $5^x = u$.
 $u^2 + u - 20 = 0$
 $(u - 4)(u + 5) = 0$
 $u = 4 \vee u = -5$
 $5^x = 4 \vee 5^x = -5$
 $x = {}^5\log(4)$
- b** $3^x - 2 = 8 \cdot \left(\frac{1}{3}\right)^x$
 $3^x - 2 = 8 \cdot \frac{1}{3^x}$
 $(3^x)^2 - 2 \cdot 3^x = 8$
 $(3^x)^2 - 2 \cdot 3^x - 8 = 0$
Stel $3^x = u$.
 $u^2 - 2u - 8 = 0$
 $(u + 2)(u - 4) = 0$
 $u = -2 \vee u = 4$
 $3^x = -2 \vee 3^x = 4$
 $x = {}^3\log(4)$
- c** $2^x = 6 - 5 \cdot \left(\frac{1}{2}\right)^x$
 $2^x = 6 - 5 \cdot \frac{1}{2^x}$
 $(2^x)^2 = 6 \cdot 2^x - 5$
 $(2^x)^2 - 6 \cdot 2^x + 5 = 0$
Stel $2^x = u$.
 $u^2 - 6u + 5 = 0$
 $(u - 1)(u - 5) = 0$
 $u = 1 \vee u = 5$
 $2^x = 1 \vee 2^x = 5$
 $x = 0 \vee x = {}^2\log(5)$
- d** $3^x + 2 \cdot \left(\frac{1}{3}\right)^{x-2} = 9$
 $3^x + 2 \cdot \left(\frac{1}{3}\right)^x \cdot \left(\frac{1}{3}\right)^{-2} = 9$
 $3^x + 18 \cdot \frac{1}{3^x} = 9$
 $(3^x)^2 + 18 = 9 \cdot 3^x$
 $(3^x)^2 - 9 \cdot 3^x + 18 = 0$
Stel $3^x = u$.
 $u^2 - 9u + 18 = 0$
 $(u - 3)(u - 6) = 0$
 $u = 3 \vee u = 6$
 $3^x = 3 \vee 3^x = 6$
 $x = 1 \vee x = {}^3\log(6)$

Bladzijde 41

$$79 \quad 3 \cdot 27^x + 2 \cdot \left(\frac{1}{3}\right)^x = 7 \cdot 3^x$$

$$3 \cdot (3^3)^x + 2 \cdot \frac{1}{3^x} = 7 \cdot 3^x$$

$$3 \cdot (3^x)^3 + 2 \cdot \frac{1}{3^x} = 7 \cdot 3^x$$

$$3 \cdot (3^x)^4 + 2 = 7 \cdot (3^x)^2$$

$$3 \cdot (3^x)^4 - 7 \cdot (3^x)^2 + 2 = 0$$

Stel $3^x = u$.

$$3u^4 - 7u^2 + 2 = 0$$

$$D = (-7)^2 - 4 \cdot 3 \cdot 2 = 25$$

$$u^2 = \frac{7+5}{6} \vee u^2 = \frac{7-5}{6}$$

$$u^2 = 2 \vee u^2 = \frac{1}{3}$$

$$u = \sqrt{2} \vee u = -\sqrt{2} \vee u = \sqrt{\frac{1}{3}} \vee u = -\sqrt{\frac{1}{3}}$$

$$3^x = \sqrt{2} \vee 3^x = -\sqrt{2} \vee 3^x = \sqrt{\frac{1}{3}} \vee 3^x = -\sqrt{\frac{1}{3}}$$

$$x = {}^3\log(\sqrt{2}) \vee 3^x = 3^{-\frac{1}{2}}$$

$$x = {}^3\log(\sqrt{2}) \vee x = -\frac{1}{2}$$

80 Voor f geldt $y = 2^x$, dus voor f^{inv} geldt $x = 2^y$.

$$x = 2^y \text{ geeft } 2^y = x$$

$$y = {}^2\log(x)$$

Dus $g(x) = {}^2\log(x)$ is de inverse van f .

Bladzijde 42

81 a $ax + b = 0$ geeft $x = -\frac{b}{a}$.

De verticale asymptoot van de grafiek is de lijn $x = -\frac{b}{a}$.

b ${}^3\log(2x+5) = 2$

$$2x+5 = 3^2$$

$$2x = 4$$

$$x = 2$$

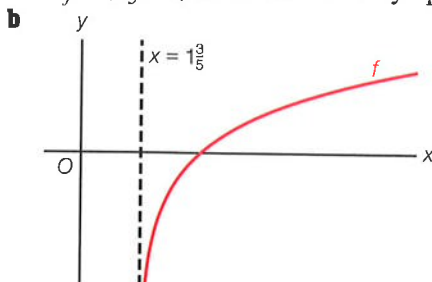
$${}^3\log(2x+5) \leq 2 \text{ geeft } -2\frac{1}{2} < x \leq 2$$

82 a $5x - 8 > 0$

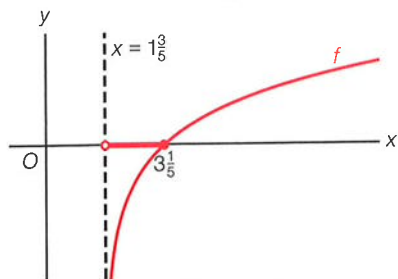
$$5x > 8$$

$$x > 1\frac{3}{5}$$

$D_f = (1\frac{3}{5}, \rightarrow)$ en de verticale asymptoot van de grafiek van f is de lijn $x = 1\frac{3}{5}$.

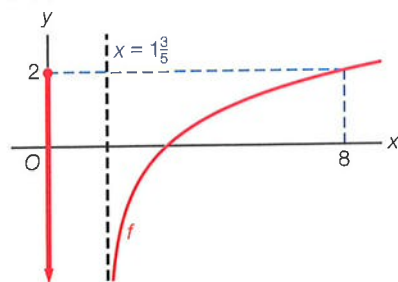


c $f(x) = 0$ geeft $-3 + {}^2\log(5x - 8) = 0$
 ${}^2\log(5x - 8) = 3$
 $5x - 8 = 2^3$
 $5x = 16$
 $x = 3\frac{1}{5}$



$f(x) \leq 0$ geeft $1\frac{3}{5} < x \leq 3\frac{1}{5}$

d $f(8) = -3 + {}^2\log(32) = -3 + 5 = 2$



Voor $x \leq 8$ is $f(x) \leq 2$.

83 a $f(x) = 5$ geeft $\frac{1}{2}\log(x + 3) = 5$

$$x + 3 = (\frac{1}{2})^5$$

$$x = -2\frac{31}{32}$$

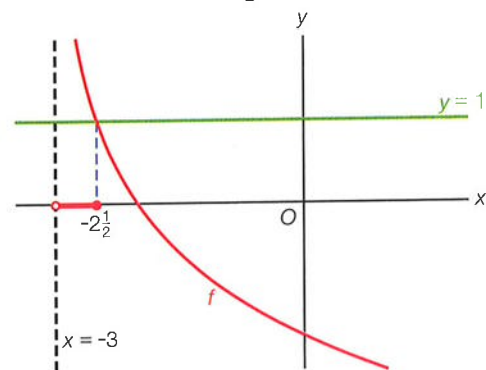
b $f(-1) = \frac{1}{2}\log(2) = -1$ en $g(-1) = {}^3\log(6)$

$$AB = {}^3\log(6) - (-1) = {}^3\log(6) + 1$$

c $f(x) = 1$ geeft $\frac{1}{2}\log(x + 3) = 1$

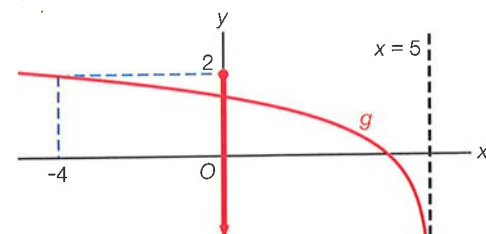
$$x + 3 = (\frac{1}{2})^1$$

$$x = -2\frac{1}{2}$$



$f(x) \geq 1$ geeft $-3 < x \leq -2\frac{1}{2}$

d $g(-4) = {}^3\log(9) = 2$



Voor $x \geq -4$ is $g(x) \leq 2$.

84 $A(4, 1)$ op de grafiek van f geeft ${}^2\log(4+c) + d = 1$.

$B(7, 3)$ op de grafiek van f geeft ${}^2\log(7+c) + d = 3$.

${}^2\log(4+c) + d = 1$ geeft ${}^2\log(4+c) = 1-d$

$$4+c = 2^{1-d}$$

$$c = 2^{1-d} - 4$$

${}^2\log(7+c) + d = 3$ geeft ${}^2\log(7+c) = 3-d$

$$7+c = 2^{3-d}$$

$$c = 2^{3-d} - 7$$

$c = 2^{1-d} - 4$ en $c = 2^{3-d} - 7$ geeft $2^{1-d} - 4 = 2^{3-d} - 7$

$$2 \cdot 2^{-d} - 4 = 2^3 \cdot 2^{-d} - 7$$

$$2 \cdot 2^{-d} - 4 = 8 \cdot 2^{-d} - 7$$

$$-6 \cdot 2^{-d} = -3$$

$$2^{-d} = \frac{1}{2}$$

$$d = 1$$

$d = 1$ geeft $c = 2^{1-1} - 4 = 2^0 - 4 = 1 - 4 = -3$

Dus $c = -3$ en $d = 1$.

Diagnostische toets

Bladzijde 46

1 a $(3a^{-5}b^4)^{-2} = 3^{-2} \cdot a^{10} \cdot b^{-8} = \frac{1}{9} \cdot a^{10} \cdot \frac{1}{b^8} = \frac{a^{10}}{9b^8}$

b $(\frac{2}{3}a^{-2}b)^{-2} = (\frac{2}{3})^{-2} \cdot a^4 \cdot b^{-2} = \frac{1}{(\frac{2}{3})^2} \cdot a^4 \cdot \frac{1}{b^2} = \frac{1}{\frac{4}{9}} \cdot a^4 \cdot \frac{1}{b^2} = \frac{9}{4} \cdot a^4 \cdot \frac{1}{b^2} = \frac{9a^4}{4b^2}$

c $3a^{1\frac{1}{3}}b^{-3} = 3a \cdot \sqrt[3]{a} \cdot \frac{1}{b^3} = \frac{3a \cdot \sqrt[3]{a}}{b^3}$

d $(a^{-\frac{1}{4}})^3 = a^{-\frac{3}{4}} = \frac{1}{a^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{a^3}}$

e $a^{-2}b^{\frac{1}{5}} = \frac{1}{a^2} \cdot \sqrt[5]{b} = \frac{\sqrt[5]{b}}{a^2}$

f $7a^{\frac{1}{3}}b^{\frac{3}{5}} = 7 \cdot \frac{1}{a^{\frac{1}{3}}} \cdot \sqrt[5]{b^3} = \frac{7 \cdot \sqrt[5]{b^3}}{\sqrt[3]{a}}$

2 a $\frac{\sqrt{x}}{x^2} = \frac{x^{\frac{1}{2}}}{x^2} = x^{\frac{1}{2}-2} = x^{-1\frac{1}{2}}$

b $x^2 \cdot \sqrt[3]{x} = x^2 \cdot x^{\frac{1}{3}} = x^{2\frac{1}{3}}$

c $\frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{\frac{2}{3}}} = x^{-\frac{2}{3}}$

3 a $y = 3x^{-3}(\frac{1}{2}x^2)^3 = 3 \cdot x^{-3} \cdot \frac{1}{8}x^6 = \frac{3}{8}x^3$

Dus $y = \frac{3}{8}x^3$.

b $y = \frac{1}{5x^2 \cdot \sqrt{x}} = \frac{1}{5} \cdot \frac{1}{x^{2\frac{1}{2}}} = \frac{1}{5}x^{-2\frac{1}{2}}$

Dus $y = \frac{1}{5}x^{-2\frac{1}{2}}$.

c $y = \frac{12}{5x^{-3}} \cdot \sqrt[5]{x^3} = \frac{12}{5} \cdot \frac{1}{x^{-3}} \cdot x^{\frac{3}{5}} = 2\frac{2}{5} \cdot x^3 \cdot x^{\frac{3}{5}} = 2\frac{2}{5}x^{3\frac{3}{5}}$

Dus $y = 2\frac{2}{5}x^{3\frac{3}{5}}$.

4 a $x^2 \cdot \sqrt[3]{x} = 128$

$$x^2 \cdot x^{\frac{1}{3}} = 128$$

$$x^{2\frac{1}{3}} = 128$$

$$x = (2^7)^{\frac{3}{7}}$$

$$x = 2^3$$

$$x = 8$$

b $(2x + 3)^{-\frac{2}{3}} = \frac{4}{9}$

$$2x + 3 = \left(\frac{4}{9}\right)^{-\frac{3}{2}}$$

$$2x + 3 = \left(\left(\frac{2}{3}\right)^2\right)^{-\frac{3}{2}}$$

$$2x + 3 = \left(\frac{2}{3}\right)^{-3}$$

$$2x + 3 = \frac{1}{\left(\frac{2}{3}\right)^3}$$

$$2x + 3 = \frac{1}{\frac{8}{27}}$$

$$2x + 3 = \frac{27}{8}$$

$$2x = \frac{3}{8}$$

$$x = \frac{3}{16}$$

c $112 - 2x^{-4} = 5x^{-4}$

$$-7x^{-4} = -112$$

$$x^{-4} = 16$$

$$x = 16^{-\frac{1}{4}}$$

$$x = (2^4)^{-\frac{1}{4}}$$

$$x = \frac{1}{2}$$

5 a $y = 0,02x^{-1\frac{3}{5}}$

$$0,02x^{-1\frac{3}{5}} = y$$

$$x^{-\frac{8}{5}} = 50y$$

$$x = (50y)^{-\frac{5}{8}}$$

$$x = 50^{-\frac{5}{8}} \cdot y^{-\frac{5}{8}}$$

$$x \approx 0,09 \cdot y^{-0,63}$$

$$\text{Dus } x = 0,09y^{-0,63}.$$

b $y = \frac{1}{4}x^2 \cdot \sqrt[3]{x} = \frac{1}{4}x^2 \cdot x^{\frac{1}{3}} = \frac{1}{4}x^{2\frac{1}{3}}$

$$\frac{1}{4}x^{2\frac{1}{3}} = y$$

$$x^{\frac{7}{3}} = 4y$$

$$x = (4y)^{\frac{3}{7}}$$

$$x = 4^{\frac{3}{7}} \cdot y^{\frac{3}{7}}$$

$$x \approx 1,81 \cdot y^{0,43}$$

$$\text{Dus } x = 1,81y^{0,43}.$$

c $y = \frac{20}{x^2 \cdot \sqrt{x}} = \frac{20}{x^{2\frac{1}{2}}} = 20x^{-2\frac{1}{2}}$

$$20x^{-2\frac{1}{2}} = y$$

$$x^{-\frac{5}{2}} = 0,05y$$

$$x = (0,05y)^{-\frac{2}{5}}$$

$$x = 0,05^{-\frac{2}{5}} \cdot y^{-\frac{2}{5}}$$

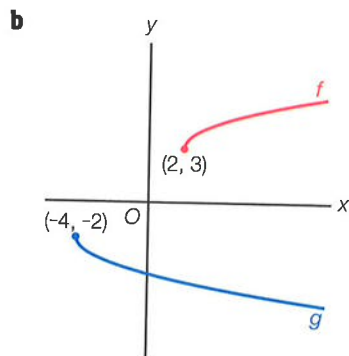
$$x \approx 3,31 \cdot y^{-0,4}$$

$$\text{Dus } x = 3,31y^{-0,4}.$$

6 $f(x) = \frac{1}{3}(x+2)^4 - 6$
 $\downarrow$ verm. x-as, -4
 $y = -1\frac{1}{3}(x+2)^4 + 24$
 $\downarrow$ translatie (3, -15)
 $g(x) = -1\frac{1}{3}(x-3+2)^4 + 24 - 15$
 oftewel $g(x) = -1\frac{1}{3}(x-1)^4 + 9$
 max. is $g(1) = 9$

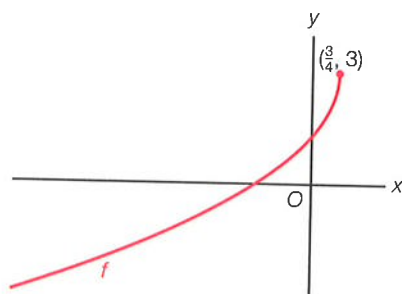
7 a $y = \sqrt{x}$
 $\downarrow$ translatie (2, 3)
 $f(x) = 3 + \sqrt{x-2}$

$y = \sqrt{x}$
 $\downarrow$ verm. x-as, -1
 $y = -\sqrt{x}$
 $\downarrow$ translatie (-4, -2)
 $g(x) = -2 - \sqrt{x+4}$



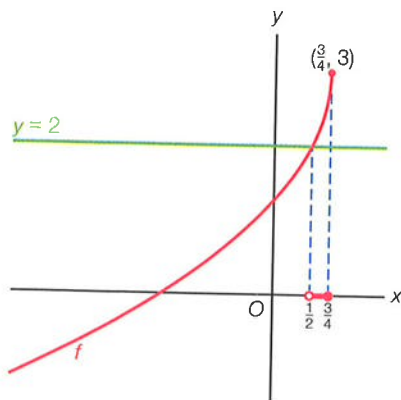
c $D_f = [2, \rightarrow)$, $B_f = [3, \rightarrow)$, $D_g = [-4, \rightarrow)$ en $B_g = \langle \leftarrow, -2 \right]$

8 a $3 - 4x \geq 0$
 $-4x \geq -3$
 $x \leq \frac{3}{4}$
 $D_f = \langle \leftarrow, \frac{3}{4} \rangle$ en het randpunt is $(\frac{3}{4}, 3)$.



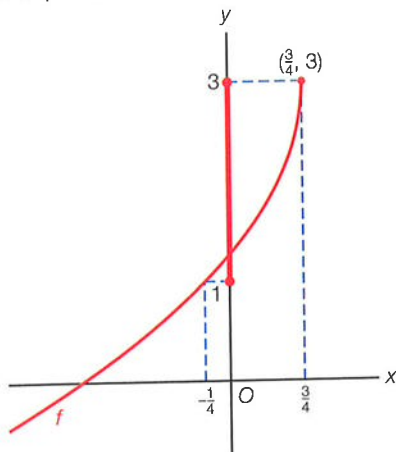
$B_f = \langle \leftarrow, 3 \rangle$

b $f(x) = 2$ geeft $3 - \sqrt{3-4x} = 2$
 $\sqrt{3-4x} = 1$
 $3 - 4x = 1$
 $-4x = -2$
 $x = \frac{1}{2}$



$f(x) > 2$ geeft $\frac{1}{2} < x \leq \frac{3}{4}$

c $f(-\frac{1}{4}) = 1$



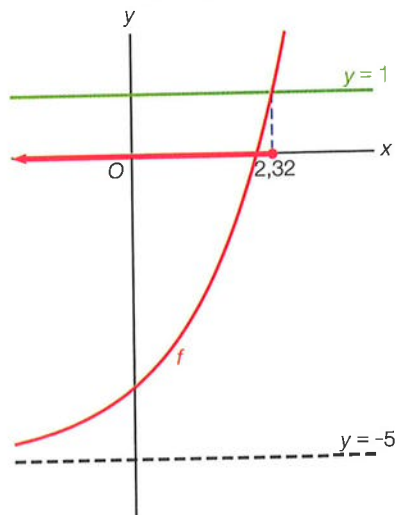
Voor $x \geq -\frac{1}{4}$ is $1 \leq f(x) \leq 3$.

9 a $N = 2\sqrt{-5t + 1}$
 $2\sqrt{-5t + 1} = N$
 kwadrateren geeft
 $4(-5t + 1) = N^2$
 $-20t + 4 = N^2$
 $-20t = N^2 - 4$
 $t = -\frac{1}{20}N^2 + \frac{1}{5}$

b $2x\sqrt{y} - 6\sqrt{x} = 1$
 $2x\sqrt{y} = 6\sqrt{x} + 1$
 kwadrateren geeft
 $4x^2y = (6\sqrt{x} + 1)^2$
 $4x^2y = 36x + 12\sqrt{x} + 1$
 $y = \frac{36x + 12\sqrt{x} + 1}{4x^2}$

Bladzijde 47

- 10 a $y = 2^x$
 ↓ verm. x-as, 0,3
 $y = 0,3 \cdot 2^x$
 ↓ translatie (-2, -5)
 $f(x) = 0,3 \cdot 2^{x+2} - 5$
- b $B_f = \langle -5, \rightarrow \rangle$
 De horizontale asymptoot is de lijn $y = -5$.
- c Voer in $y_1 = 0,3 \cdot 2^{x+2} - 5$ en $y_2 = 1$.
 De optie snijpunt geeft $x \approx 2,32$.



$f(x) \leq 1$ geeft $x \leq 2,32$

11 a $y = 2^x \cdot 2^{-4x+3} = 2^x \cdot 2^{-4x} \cdot 2^3 = 2^{-3x} \cdot 8 = 8 \cdot (2^{-3})^x = 8 \cdot (\frac{1}{8})^x$

Dus $y = 8 \cdot (\frac{1}{8})^x$.

b $y = 108 \cdot 3^{4x-3} = 108 \cdot 3^{4x} \cdot 3^{-3} = 108 \cdot (3^4)^x \cdot \frac{1}{27} = 4 \cdot 81^x$

Dus $y = 4 \cdot 81^x$.

c $y = \frac{250}{5^{-2x+3}} = \frac{250}{5^{-2x} \cdot 5^3} = \frac{250}{5^{-2x} \cdot 125} = 2 \cdot 5^{2x} = 2 \cdot (5^2)^x = 2 \cdot 25^x$

Dus $y = 2 \cdot 25^x$.

12 a $5^{x-1} = 125\sqrt{5}$

$5^{x-1} = 5^3 \cdot 5^{\frac{1}{2}}$

$5^{x-1} = 5^{3\frac{1}{2}}$

$x-1 = 3\frac{1}{2}$

$x = 4\frac{1}{2}$

b $3^{2x-5} = (\frac{1}{27})^x$

$3^{2x-5} = (3^{-3})^x$

$3^{2x-5} = 3^{-3x}$

$2x-5 = -3x$

$5x = 5$

$x = 1$

c $2 \cdot 4^{2x-1} - 3 = 61$

$2 \cdot 4^{2x-1} = 64$

$4^{2x-1} = 32$

$(2^2)^{2x-1} = 2^5$

$2^{4x-2} = 2^5$

$4x-2 = 5$

$4x = 7$

$x = 1\frac{3}{4}$

d $9^{x-1} = 27^{x+1}$

$(3^2)^{x-1} = (3^3)^{x+1}$

$3^{2x-2} = 3^{3x+3}$

$2x-2 = 3x+3$

$-x = 5$

$x = -5$

e $2^{x+2} + 2^{x-1} = 36$

$2^x \cdot 2^2 + 2^x \cdot 2^{-1} = 36$

$4 \cdot 2^x + \frac{1}{2} \cdot 2^x = 36$

$4\frac{1}{2} \cdot 2^x = 36$

$2^x = 8$

$2^x = 2^3$

$x = 3$

f $2^{x^2} = (\frac{1}{8})^x$

$2^{x^2} = (2^{-3})^x$

$2^{x^2} = 2^{-3x}$

$x^2 = -3x$

$x^2 + 3x = 0$

$x(x+3) = 0$

$x = 0 \vee x = -3$

13 a $f(x) = g(x)$ geeft $2^{x+2} - 3 = 6 - 2^{x-1}$

$2^x \cdot 2^2 - 3 = 6 - 2^x \cdot 2^{-1}$

$4 \cdot 2^x - 3 = 6 - \frac{1}{2} \cdot 2^x$

$4\frac{1}{2} \cdot 2^x = 9$

$2^x = 2$

$x = 1$

$f(x) \geq g(x)$ geeft $x \geq 1$

b $g(4) = 6 - 2^3 = 6 - 8 = -2$

Voor $x \leq 4$ is $-2 \leq g(x) < 6$.

c $f(x) + g(x) = 2^{x+2} - 3 + 6 - 2^{x-1} = 2^x \cdot 2^2 - 3 + 6 - 2^x \cdot 2^{-1} = 4 \cdot 2^x + 3 - \frac{1}{2} \cdot 2^x = 3\frac{1}{2} \cdot 2^x + 3$

Het bereik van $f(x) + g(x)$ is $\langle 3, \rightarrow \rangle$.

Dus de vergelijking $f(x) + g(x) = p$ heeft geen oplossing voor $p \leq 3$.

14 a ${}^3\log(3\sqrt{3}) = {}^3\log(3^1 \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{1\frac{1}{2}}) = 1\frac{1}{2}$

b ${}^2\log(\frac{1}{4}\sqrt{8}) = {}^2\log(2^{-2} \cdot \sqrt{2^3}) = {}^2\log(2^{-2} \cdot 2^{\frac{3}{2}}) = {}^2\log(2^{-\frac{1}{2}}) = -\frac{1}{2}$

c ${}^2\log(\frac{1}{16} \cdot \sqrt[3]{2}) = {}^2\log(2^{-4} \cdot 2^{\frac{1}{3}}) = {}^2\log(2^{-\frac{11}{3}}) = -3\frac{2}{3}$

15 a ${}^4\log(2x-3) = 2$

$2x-3 = 4^2$

$2x-3 = 16$

$2x = 19$

$x = 9\frac{1}{2}$

b $\frac{1}{2}\log(x-3) = -4$

$x-3 = (\frac{1}{2})^{-4}$

$x-3 = 16$

$x = 19$

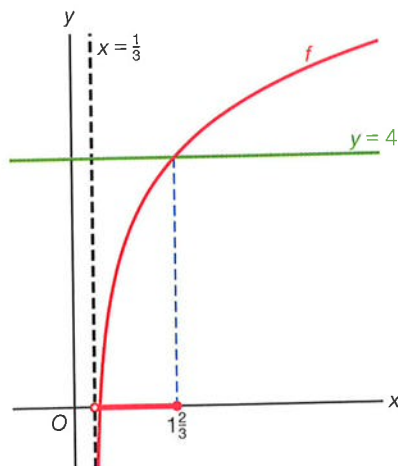
$$\begin{aligned} \text{c } 5 + 3 \cdot {}^2\log(x) &= 20 \\ 3 \cdot {}^2\log(x) &= 15 \\ {}^2\log(x) &= 5 \\ x &= 2^5 \\ x &= 32 \end{aligned}$$

$$\begin{aligned} 16 \text{ a } 7^{x-3} &= 20 \\ x - 3 &= {}^7\log(20) \\ x &= 3 + {}^7\log(20) \end{aligned}$$

$$\begin{aligned} \text{b } 4^x - 2^{x+4} &= 80 \\ (2^2)^x - 2^x \cdot 2^4 &= 80 \\ (2^x)^2 - 16 \cdot 2^x - 80 &= 0 \\ \text{Stel } 2^x &= u. \\ u^2 - 16u - 80 &= 0 \\ (u + 4)(u - 20) &= 0 \\ u = -4 \vee u &= 20 \\ 2^x = -4 \vee 2^x &= 20 \\ x &= {}^2\log(20) \end{aligned}$$

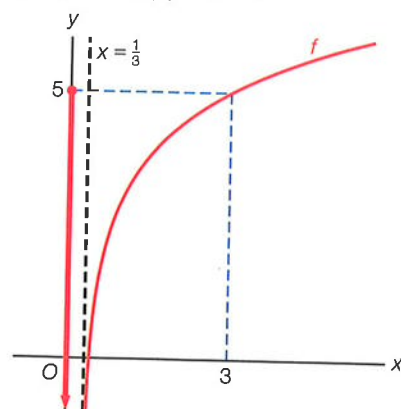
$$\begin{aligned} \text{c } 5^{x+1} - 6 \cdot \left(\frac{1}{5}\right)^{x-1} &= 25 \\ 5^x \cdot 5 - 6 \cdot \left(\frac{1}{5}\right)^x \cdot \left(\frac{1}{5}\right)^{-1} &= 25 \\ 5 \cdot 5^x - 30 \cdot \frac{1}{5^x} &= 25 \\ 5 \cdot (5^x)^2 - 30 &= 25 \cdot 5^x \\ (5^x)^2 - 5 \cdot 5^x - 6 &= 0 \\ \text{Stel } 5^x &= u. \\ u^2 - 5u - 6 &= 0 \\ (u + 1)(u - 6) &= 0 \\ u = -1 \vee u &= 6 \\ 5^x = -1 \vee 5^x &= 6 \\ x &= {}^5\log(6) \end{aligned}$$

$$\begin{aligned} 17 \text{ a } 3x - 1 > 0 \text{ geeft } x &> \frac{1}{3} \\ D_f = \left(\frac{1}{3}, \rightarrow\right) \text{ en de verticale asymptoot van de grafiek is de lijn } x &= \frac{1}{3}. \\ f(x) = 4 \text{ geeft } {}^2\log(3x - 1) + 2 &= 4 \\ {}^2\log(3x - 1) &= 2 \\ 3x - 1 &= 2^2 \\ 3x - 1 &= 4 \\ 3x &= 5 \\ x &= 1\frac{2}{3} \end{aligned}$$



$$f(x) \leq 4 \text{ geeft } \frac{1}{3} < x \leq 1\frac{2}{3}$$

b $f(3) = {}^2\log(8) + 2 = 5$



Voor $x \leq 3$ is $f(x) \leq 5$.